# Financial Constraints and Firm Size: Micro-Evidence and Aggregate Implications[*]

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**Abstract**

Using a unique dataset covering the universe of Portuguese firms, their credit situation, and bank relationship we show that firms across the entire size distribution exhibit a positive elasticity of loans and capital to credit supply shocks. This finding is counterfactual to basic theory that posits large firms as unconstrained and not reacting to credit supply shocks. Incorporating a richer, empirically supported, productivity process into a standard heterogeneous firms model generates large constrained firms and consequently a joint distribution of size and credit elasticities in line with the data. The elevated capital share and sensitivity to financial shocks of the largest decile of constrained firms explains about one-third of the response of output to a financial shock.

**Keywords:** Firm Size, Business Cycle, Financial Constraints

**JEL Codes:** E62, E22, E23

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# 1 Introduction

Macroeconomic research has long emphasized the propagation of aggregate shocks through financial channels and their interaction with firm-specific characteristics. A notable contribution in this area is the work by Gertler & Gilchrist (1994), who identify firm size as a proxy for financial constraints, suggesting that smaller firms are less liquid, more risky and dependent on external finance compared to their larger counterparts. Many modern heterogeneous firm models with financial frictions also mirror this argument, generating a strong dichotomy between small, constrained and large, unconstrained firms. Yet, despite recent advances in the empirical literature, the strength of this association remains ambiguous (see e.g. Crouzet & Mehrotra, 2020). This is unfortunate, as the relation between size and the strength of financial constraints is consequential for both macroeconomic outcomes and policy design.

This paper provides new insights into this critical relationship and its implications for aggregate outcomes. First, we argue that the elasticity of credit to bank supply shocks offers an important metric for assessing financial frictions across the firm size distribution. Using granular bank-firm-level data from Portugal and exploiting exogenous variation in bank supply, we subsequently estimate this elasticity across different firm size deciles. Second, we show that a financial frictions model that incorporates both transitory and permanent productivity components can replicate the observed elasticity distribution. Finally, we use this calibrated model to evaluate the role of size-constraint distributions in shaping aggregate outcomes. Our analysis emphasizes, to the best of our knowledge for the first time, the importance of size-dependent elasticities to bank supply shocks as key statistics for understanding the aggregate consequences of financial frictions.

Our first main finding in the paper is about the empirical estimation of credit elasticities. This estimation is grounded in a simple theoretical framework: firms that are not financially constrained should exhibit little or no elasticity of debt issuance when exposed to exogenous credit supply shocks, as the price or quantity constraint is not binding. By contrast, constrained firms are expected to show a positive elasticity. This reasoning applies to both convex loan supply schedules and strict collateral constraints, providing a comprehensive framework for linking the elasticity to the degree of financial constraint.

We test this hypothesis using data from the Bank of Portugal's credit registry, which tracks monthly credit information for individuals and non-financial corporations from 2006 to 2017. Matched with firm and bank balance sheets, and applying the identification strategy of Amiti & Weinstein (2018), we separate loan growth into bank supply and firm demand components and analyze the elasticity of credit and investment to the bank supply component.

Our findings reveal that the credit elasticity to exogenous bank supply shocks is significant and only mildly decreasing across firm size deciles. Importantly, this elasticity remains positive even for the largest firms, implying that firms of all sizes respond to changes in bank credit supply. On average, a 1% increase in bank credit supply leads to a 0.37% rise in firm credit. Furthermore, to ensure that these bank supply variations also affect real outcomes, we analyze the effects on investment and observe qualitatively similar effects. Overall, when viewed through the lens of our simplified theoretical framework, our first finding suggests that financial constraints affect firms across the entire size distribution.[1]

The second main finding in this paper relates to the aggregate implications of the empirically observed size-elasticity profile. We employ a heterogeneous firm model with both transitory and permanent productivity shocks in order to assess the aggregate effects. The inclusion of permanent productivity heterogeneity allows the model to generate a joint size-elasticity distribution that closely aligns with the empirical data. This variation in permanent productivity introduces large, persistent differences in firm size and financial constraint episodes, enabling the model to account for the presence of large, financially constrained firms.[2]

We then analyze the macroeconomic implications of the presence of large constrained firms. Our results indicate that financial shocks are significantly amplified by the presence of these firms, with the largest 10% of constrained firms responsible for nearly one-third of the aggregate responses in output and investment. Two factors contribute to this amplification: the elasticity of constrained firms to shocks remains constant across the size distribution, and large constrained firms hold a disproportionate share of the economy's productive capital. Together, these elements explain why aggregate output and investment responses are so heavily influ-

[1]Our results align with evidence from Peek & Rosengren (2000), Khwaja & Mian (2008), Chaney et al. (2012), Chodorow-Reich (2014), Farre-Mensa & Ljungqvist (2016), and Herreno (2023), who show credit supply shocks affect firm credit growth and real outcomes.

[2]We do find empirical validation for our permanent productivity component in Portugal. This is in line with the recent evidence on permanent heterogeneity among firms in Pugsley et al. (2021) and outlined in our Appendix.

enced by the behavior of large constrained firms.

We proceed to examine whether the presence of large constrained firms leads to significant differences between incomplete markets and complete markets real business cycle (RBC) models in response to a positive total factor productivity (TFP) shock. Incomplete markets models typically generate constrained firms only at the lower end of the size distribution, resulting in output and investment responses comparable to those in RBC models. However, when large constrained firms, which own a significant portion of capital, are accounted for, the incomplete markets model's response to persistent TFP shocks is 15% smaller than that of the RBC model, as constrained firms are unable to fully adjust to the shock.

Lastly, we explore a potential policy that could limit the negative effects of a financial shock. Despite large constrained firms accounting for about one-third of the output response to a negative financial shock, we find it is still optimal to subsidize small firms for two reasons: 1) financial constraints are still more prevalent at the bottom of the size distribution, so the capital elasticity is still higher at the bottom than at the top of the size distribution; and 2) small firms have higher growth potential, which causes output to recover faster.

Overall, our paper highlights the importance of size-dependent credit and capital elasticities as critical moments for matching the joint distribution of firm size and financial constraints. Ignoring large constrained firms may lead to a significant underestimation of the transmission and amplification of financial shocks on aggregate output.

**Literature** Our work follows a large literature in macroeconomics that has analyzed heterogeneous firms and financial frictions both theoretically and empirically.

First, our work is connected to the empirical literature on the cyclicality of constrained firms and the debate on how to identify these firms. Gertler & Gilchrist (1994) provide evidence from the QFR dataset for the financial accelerator by analyzing the different cyclical behavior of small and large manufacturing firms, using size as a proxy for financial constraints. This is supported by Sharpe (1994), who finds that smaller firms exhibit more cyclical employment growth, as they are less able to hoard labor. Gilchrist & Himmelberg (1995) further show that investment is more sensitive to cash flow for firms with limited access to capital markets. However, recent work by Crouzet & Mehrotra (2020) challenges the connection between size and financial frictions, showing that size-related cyclicality is only significant for the very largest firms, while the bottom 99.5% exhibit non-significant differences. This, together with the insignificance of common proxies for financial constraints, suggests that financial factors may not be as important across the size distribution. Relatedly, Farre-Mensa & Ljungqvist (2016) argue that standard constraint measures don't match firm behavior, a finding also confirmed by Bodnaruk et al. (2015) through text analysis of financial reports. Additionally, recent studies propose other relevant proxies, such as age (Cloyne et al., 2023), leverage and distance to default (Ottonello & Winberry, 2020), liquidity (Jeenas, 2023), and MRPKs (González et al., 2024), to capture firms' sensitivity to monetary policy.

Our paper, by making use of detailed firm-level credit data, contributes to this literature by reiterating that size is indeed an imperfect proxy for financial constraints. Moreover, with our test that uses bank-firm-level loan variation, we argue that more detailed financial data is needed to correctly identify the distribution of financially constrained firms. We also provide evidence in support of a broader financial accelerator mechanism that is only weakly size dependent.

Second, we contribute to the expanding research on models with heterogeneous firms and financial frictions in the context of business cycle analysis.[3] A key early contribution to this literature, by Cooley & Quadrini (2001), closely aligns with our model. They extend a standard Hopenhayn (1992) heterogeneous firm framework by incorporating financial frictions and persistent shocks, which allows them to capture key empirical patterns—namely, that smaller firms (conditional on age) and younger firms (conditional on size) are more dynamic, exhibiting higher rates of job creation and destruction, growth, volatility, and exit. Building on this, Clymo & Rozsypal (2019), using administrative data, also find that young and small firms are almost twice as cyclical as large firms. Similarly, Pugsley et al. (2021) underscore the role of ex-ante heterogeneity in shaping the firm size distribution and the observed decline in firm dynamism over recent decades. A contribution that bridges the connection between financial frictions and permanent heterogeneity is Mehrotra & Sergeyev (2020). They argue that financial frictions played a relatively minor role in the rise of unemployment during the Great Recession.

[3]There is also a significant body of growth literature that focuses on financial frictions on the supply side. Related contributions include Buera et al. (2011), Buera & Shin (2013), and Moll (2014).

Instead, they suggest that shocks affected both constrained and unconstrained firms, leading to widespread employment reductions. In contrast, Khan & Thomas (2013) and Jermann & Quadrini (2012) highlight the critical role of financial frictions and financial shocks in driving aggregate dynamics, offering a different perspective. [4] We relate to this literature by showing how firm heterogeneity, especially regarding size and financial constraints proxied by credit elasticities, amplifies these effects on aggregate outcomes.

Finally, our work is related to the effects of credit shocks on firms. Buera & Moll (2015) emphasize that understanding firm-level heterogeneity is crucial for assessing the aggregate implications of credit disruptions. Micro-level data are key to identifying the mechanisms through which such shocks propagate. Studies like Peek & Rosengren (2000), and Khwaja & Mian (2008) demonstrate that reductions in bank credit supply lead to declines in credit and employment for firms connected to affected banks. Building on this, Herreno (2023) models the aggregate impacts of these firm-level shocks, revealing large effects on output. Our contribution complements this literature by estimating size-dependent effects of credit supply shocks. Our empirical analysis closely aligns with this literature, but we estimate differential elasticities based on a firm's position in the size distribution.

**Outlook** The paper is structured as follows. Section 2 defines and develops the simple framework that links the elasticity of debt to credit supply shocks, depending on whether a firm is financially constrained. Section 3 presents the data and the main empirical analysis. In section 4 we set out the model to incorporate and account for the distribution of constrained firms and in section 5 we discuss model predictions of aggregate effects. Finally, section 6 concludes.

## 2 Financial constraints: a simple framework

In this section, we develop a simple framework that connects the elasticity of debt to credit supply shocks, depending on whether a firm is financially constrained. While our empirical analysis will estimate firms' credit elasticity to bank supply shocks, it is first essential to understand why this elasticity may vary across firms based on their financial situations. To build this un-

[4] On the monetary policy side, González et al. (2024) show that loose monetary policy disproportionately benefits high-productivity firms, improving resource allocation in general equilibrium.

derstanding, we begin by formalizing two widely used definitions of financial constraints from the literature and compare how the elasticity of constrained and unconstrained firms differs under each definition. Importantly, under both definitions, the elasticity of credit serves as a critical measure for determining whether a firm is financially constrained. Firms with a higher elasticity in response to credit supply shocks are more likely to be constrained, while those with lower elasticity tend to be less financially constrained.

## 2.1 Supply curve curvature

Seminal contributions such as Stiglitz & Weiss (1981), Almeida & Campello (2001), and Whited & Wu (2006) define financial constraints in terms of the location of a firm along a convex credit supply curve. Firms positioned on the inelastic part of the curve, where obtaining additional credit comes at a higher marginal cost, are more likely to be financially constrained. This concept is illustrated in Figure 1a, which depicts two distinct demand schedules: one with a low demand for credit, labeled $D_1$, and another with a higher demand for credit, labeled $D_2$. According to the definition above, the firm with demand schedule $D_2$ is financially constrained, as it operates on the inelastic portion of the supply curve $S$.

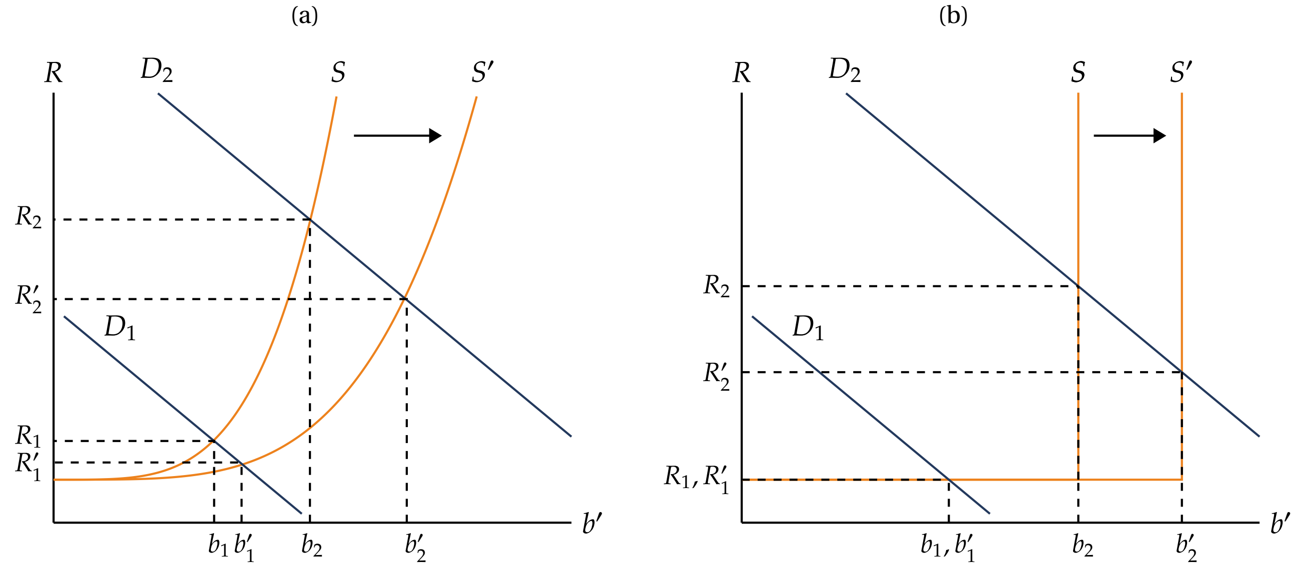
When the supply of credit increases, shifting the supply curve outward from $S$ to $S'$, the equilibrium level of credit rises significantly for firms with demand $D_2$. This results in a larger elasticity of credit for these firms, as shown by the greater change in equilibrium credit levels.

In contrast, firms that obtain credit closer to the risk-free rate, represented by demand curve $D_1$, show little response to the supply shock as they could already obtain credit at a relatively low rate. Consequently, their elasticity is closer to zero, as indicated by the smaller increase in credit levels after the shock. This framework demonstrates how financially constrained firms are more responsive to credit supply shifts, while firms closer to the risk-free rate exhibit a lower sensitivity.

## 2.2 Collateral constraint

Another definition of financial constraints focuses on collateral limitations. Works such as Kiyotaki & Moore (1997), Khan & Thomas (2013), and Catherine et al. (2022) define a firm as finan-

Figure 1: Supply changes with decreasing elasticity of supply and collateral constraint




*Note.* The orange lines represent the supply schedule whilst the blue lines represent demand schedules for debt. Panel (a) depicts a schematic supply expansion for an upward sloping supply curve whilst panel (b) depicts a supply expansion with a quantity collateral constraint.

cially constrained when its borrowing capacity is limited by the value of its collateral. Figure 1b illustrates this concept. Up to the value of the firm's collateral the firm can borrow at the risk-free rate, as the loan is fully secured by the collateral. In this region, the credit supply curve is flat, meaning the firm can borrow more without facing higher marginal costs. However, once the firm's debt reaches the value of its collateral, the supply curve becomes inelastic, and even if the firm is willing to pay a higher interest rate, it cannot access additional credit.

To better understand the impact of credit supply shocks under this definition, we again consider a shift of the supply curve from $S$ to $S'$, as shown in Figure 1b, along with the two demand schedules. Again, firms operating along the demand schedule $D_1$ represent unconstrained firms as they are located on the horizontal, elastic portion of the supply curve. For these firms, neither the cost of debt nor the equilibrium level of credit changes in response to a supply shift from $S$ to $S'$, resulting in an elasticity of credit that remains at zero.

In contrast, firms located on the demand schedule $D_2$, which are constrained by the value of their collateral, are positioned on the vertical, inelastic portion of the supply curve. As the credit supply shifts outward, these firms will borrow as much as their collateral allows until the constraint binds again. Consequently, constrained firms exhibit a significant response to the credit supply shock, resulting in an elasticity of credit close to one.

Although this collateral-based definition of financial constraints is more extreme, classifying firms as either having an elasticity of zero or one, it similarly highlights how constrained firms—those with high demand for credit—are much more sensitive to supply shocks compared to unconstrained firms. This framework reinforces the idea that financially constrained firms respond more strongly to shifts in credit supply, while firms operating nearer the risk-free rate, such as those on $D_1$, exhibit little to no sensitivity.

# 3 Empirical Evidence

In this section, we first describe the construction and cleaning of the dataset, followed by the explanation of our main econometric analysis on the differential elasticity of firms to credit supply shocks across the size distribution. Finally, we provide a short discussion of our evidence.

## 3.1 Data

We draw on a unique combination of datasets that cover the Portuguese economy between 2006 and 2017, all managed by the Bank of Portugal Microdata Research Laboratory. The *Informação Empresarial Simplificada* (IES) Central Balance Sheet Database is based on annual accounting data of individual firms. Portuguese firms have to fill out mandatory financial statements in order to comply with their statutory obligation. Consequently, this dataset covers virtually the entire population of non-financial corporations in Portugal from 2006 onwards. We combine this dataset with the Central Credit Responsibility (CCR) dataset which contains monthly information on actual and potential credit above 50 Euros extended to individuals and non-financial corporations, reported by all financial institutions in Portugal.[5] Actual credit includes loans that are truly taken up, such as mortgages, consumer loans, overdrafts and others. Potential credit encompasses all irrevocable commitments to the subject that have not materialized into actual credit, such as credit available on credit lines, credit cards, pledges granted by participants and other credit facilities.[6] We then merge these two databases on the firm-level. Moreover, we also

[5]Given that the firm balance sheet data is annual we consider the month in which the balance sheet data is reported. Results were robust to shifting and averaging the monthly credit data.

[6]Further details on the credit information used are also documented in appendix A.

Table 1: Firm employment distributions in 2017: Portugal and the US

| Size indicator | Number of firms | | Employment Share | |
|---|---|---|---|---|
| | Portugal | USA | Portugal | USA |
| < 5 employees | 48.2% | 61.7% | 6.3% | 4.6% |
| 5 - 9 employees | 23.6% | 16.8% | 9.0% | 5.2% |
| 10 - 19 employees | 14.2% | 10.5% | 10.9% | 6.6% |
| 20 - 49 employees | 9.2% | 6.9% | 15.9% | 9.7% |
| 50 - 99 employees | 2.7% | 2.2% | 10.6% | 6.9% |
| 100 - 499 employees | 1.9% | 1.5% | 21.2% | 14.1% |
| > 500 employees | 0.3% | 0.3% | 26.2% | 52.9% |

*Note.* This table provides the employment distribution of firms both in Portugal and the US. The US numbers are sourced from the 2017 Statistics of US Businesses (SUSB) Annual Data Tables available from the U.S. Census Bureau. The numbers for Portugal are from our cleaned dataset.

add the Monetary Financial Institutions Balance Sheet Database in order to gain information on the balance sheets of banks that extend credit to non-financial institutions. We merge this dataset at the firm level using the bank identifier and the share of loans extended to one firm to create our final dataset.

Similar to Buera & Karmakar (2022), who use the same dataset, we restrict the set of firms in this panel dataset to those with at least five consecutive observations and to firms which are in business at the time of reporting. Furthermore, we only consider privately or publicly held firms and drop firms with overall credit amounts of less than 10,000 Euros. Descriptive statistics for the relevant variables can be found in Table A1 in Appendix A. The final sample of firms represents around 50% of total employment in Portugal.

To provide a meaningful comparison between our final dataset of Portuguese firms and the U.S. firm distribution, we include Table 1. In this table, we compare the firm employment distribution in our sample of Portuguese firms and in the United States in 2017. Both firm distributions are skewed, with both countries having only 0.3% of firms with more than 500 employees. This small fraction of firms disproportionately accounts for a large share of employment. However, while in Portugal, 0.3% of firms account for 26.2% of total employment, in the U.S., the distribution is even more skewed, with the same 0.3% of firms accounting for more than 50% of total employment.

## 3.2 Credit Supply Shocks and Debt Issuance

The main empirical strategy to test for financial frictions rests on the decomposition of loan growth into demand and supply shocks by Amiti & Weinstein (2018). In particular, they show that one can recover idiosyncratic bank lending and firm borrowing shocks from the loan variation between the two parties. The methodology relies on a linear model that separates loan growth rates into bank-time fixed effects and firm-time fixed effects.[7] In our setting we decompose the total available credit that a firm can access from bank $b$ into these two components and then weigh them according to the importance of the bank in the total loan portfolio of firm $i$. This implies the yearly loan supply and demand shocks are aggregated on the idiosyncratic firm level.

**Average elasticities** We then use these identified loan supply shocks to test for the presence of financial constraints across the distribution. First, to understand the average elasticity of credit to bank supply shocks we run the following regression:

$$g_{ijt} = \beta\epsilon_{it}^{b} + \Gamma X_{ijt} + \varrho g_{ijt-1} + \alpha_i + \alpha_{jt} + u_{ijt}, \tag{1}$$

where $i$ identifies a firm, $j$ the respective industry and $t$ identifies a year. The dependent variable $g_{ijt}$ is the year-on-year log change in total outstanding debt of firm $i$ in industry $j$. $\epsilon_{it}^{b}$ signifies the bank supply shock weighted by the respective bank share at the firm level. Finally, we incorporate firm fixed effects ($\alpha_i$) and industry-year fixed effects ($\alpha_{jt}$), along with control variables at both the firm level—such as liquidity and leverage ratios—and the bank level, including capital and liquidity ratios and the share of sovereign bonds to total assets.These controls help account for potential differences in firm and bank credit demand and supply curves, ensuring that the effect of the shock is properly identified and not driven by other factors.

The coefficient of interest here will be $\beta$ as it measures the average elasticity of firms total change in debt to a bank supply shock. The basic theory of financial frictions put forward in Section 2 can give a good guidance what one would expect here. A coefficient close to zero would imply only a few firms in the economy face financial constraints as only those firms

[7] For more details, please see Amiti & Weinstein (2018).

Table 2: Average debt elasticities to bank supply shocks

| | (1) | (2) |
|---|---|---|
| Bank shock | 0.372*** | 0.462*** |
| | (0.0140) | (0.0146) |
| Firm FE | Yes | Yes |
| Industry × Year FE | Yes | Yes |
| Clustering | Firm | Firm |
| Bank and Firm Controls | No | Yes |
| Observations | 1 050 471 | 984 660 |

*Note.* This table provides the average elasticities of debt to a one unit exogenous increase in credit supply by the bank. The dependent variable is the log change in real total credit. The first column reports the average elasticity without extra controls, whilst the second one reports the one with additional bank and firm controls. Standard errors are reported in parenthesis.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

would take up the extra available credit. On the other hand, an elasticity close to one would imply that financial constraints are indeed quite prevalent over the economy.

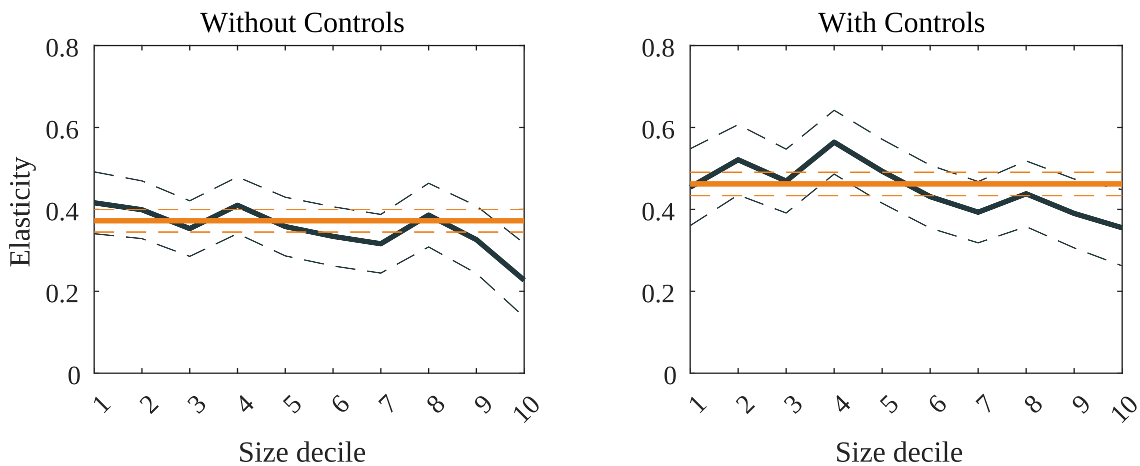
Columns (1) and (2) in Table 2 show the result for the main regression on the average effect, where the latter controls for the firm and bank level control variables. Both estimated parameters are positive and significant. For example, in the version without additional controls, a 1% increase in bank supply leads, on average, to a 0.372% increase in actual credit taken up by the firm. This result holds when winsorizing the shocks at the 1% and 99% level and controlling for credit demand shocks as well as shown in panel (iv) in Figure 2 in Appendix B. We interpret these results as evidence that, on average, firms are considerably affected by financial frictions in the Portuguese economy.

**Size-dependent credit elasticities** Next, we document the differential elasticity total debt to a bank supply shock across the size distribution by running the following regression:

$$g_{ijt} = \sum_{l \in \mathcal{L}} (\alpha_l + \beta_l \epsilon_{it}^{b}) \mathbf{1}_{i \in \mathcal{S}_t^{(l)}} + \varrho g_{ijt-1} + \Gamma X_{it} + \alpha_i + \alpha_{jt} + u_{ijt}, \tag{2}$$

where the set $\mathcal{S}_t^{(l)}$ is the $l$th size group, e.g. all firms above the 80th but below the 90th percentile. We include ten deciles of size groups and test differential elasticities across all of these groups. The rest of the variables and coefficients are defined as before.

Figure 2: Average and size dependent elasticity of credit to credit supply shocks




*Note.* The orange line reports the estimate for the average elasticity whilst the blue line depicts the estimate over the different size deciles. Dashed lines represent the 95% confidence intervals. Left panel, results without firm and bank controls. Right panel, results when including firm and bank controls. Size indicator: real total assets

Again, the set of coefficients of interest is the set $\{\beta_l\}_{l\in\mathcal{L}}$ as it measures the (potentially) differential elasticity of credit with respect to exogenous supply shocks over the distribution. A priori, if financial frictions only affect the small firms one would expect a strongly declining profile of elasticities and near zero at the top of the distribution. Vice versa, if financial frictions are pervasive, the elasticities should remain relatively constant over all ten deciles.

Figure 2 depicts the average plus the ten estimated elasticities across the deciles of the size distribution with the right panel plotting the elasticity controlling for a number of firm and bank variables. Our size indicator here is real total assets. As is evident, the elasticity stays high and relatively close to the sample average across all ten deciles, except for the top decile. However, even at the top 10 percent of firms an increase of 1% in exogenous credit supply leads to an increase of 0.227% in total credit. We take this as indirect, but compelling, evidence that financial constraints seem to be present across the size distribution.

In panels (i) to (iv) of Figure 2 in Appendix B, we present the results from various robustness checks on this empirical result. We run the same regressions as above, using different size indicators, including real turnover, lagged real total assets, and the number of employees. Across all three alternative size indicators, the qualitative results hold—large firms continue to show a positive and significant response in loan growth to the bank supply shock. Additionally, in panel (iv), we test whether the results are affected by outliers or credit demand shocks. We win-

Table 3: Average capital elasticities

| | (1) | (2) |
|---|---|---|
| Bank shock: Global credit | 0.126*** | 0.138*** |
| | (0.0126) | (0.0137) |
| Firm FE | Yes | Yes |
| Industry × Year FE | Yes | Yes |
| Clustering | Firm | Firm |
| Bank and Firm Controls | No | Yes |
| Observations | 1 050 471 | 984 660 |

*Note.* This table provides the average elasticities of capital on a one unit increase in bank supply. The dependent variable is the change in log capital. The first column reports the average elasticity without extra controls, whilst the second one reports the one with additional bank and firm controls. Standard errors are reported in parenthesis. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

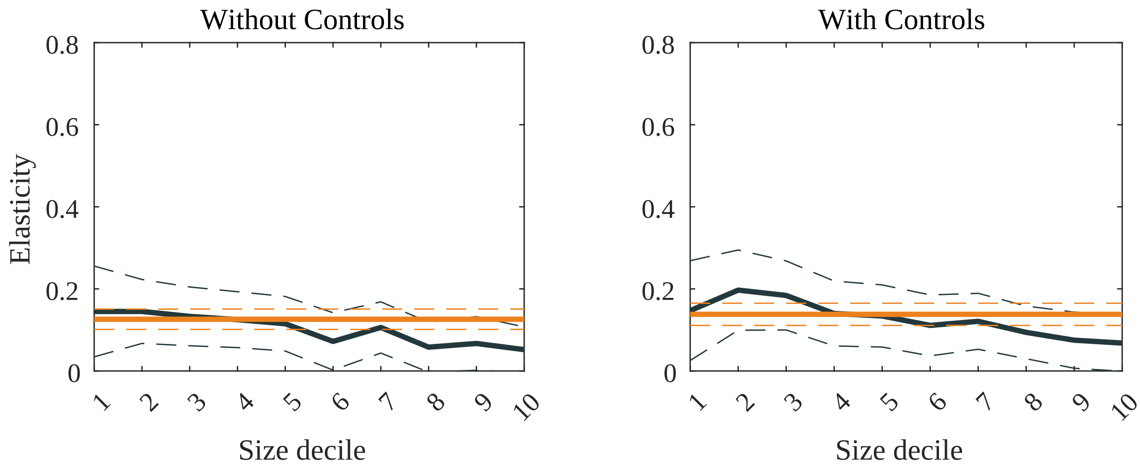
sorize the data by excluding the smallest and largest 1% of shocks. To ensure that the elasticity of credit supply shocks is independent of demand effects, we also control for credit demand shocks, estimated using the methodology of Amiti & Weinstein (2018), and interact them with the size deciles to account for size-dependent elasticity to credit demand shocks. The results remain robust to both winsorizing the shocks and controlling for size-dependent credit demand elasticities.

**Investment elasticities** So far, we have demonstrated the effect of credit supply shocks on credit growth. The next crucial question is whether these shocks translate into real economic outcomes. To explore this, we now assess their impact on firm-level investment, specifically examining whether the investment elasticity varies with firm size.

First, we extend our analysis by estimating equation (1), using the log change in physical capital as the dependent variable. Columns (1) and (2) of Table 3 present the results for this regression on the average effect, with the second specification including firm and bank-level controls. Both estimated elasticities are positive and significant. Specifically, a 1% increase in bank supply leads, on average, to a 0.126% increase in investment when firm and bank controls are excluded.

Next, we estimate equation (2) for the change in physical capital to test for differential elasticities across firm sizes. Figure 3 illustrates the elasticity by size deciles, with the left panel showing results without firm and bank controls, and the right panel including these controls.

Figure 3: Average and size dependent elasticity of capital to credit supply shocks




*Note.* The orange line reports the estimate for the average elasticity whilst the blue line depicts the estimate over the different size deciles. Dashed lines represent the 95% confidence intervals. Left panel, results without firm and bank controls. Right panel, results when including firm and bank controls. Size indicator: real total assets.

Similar to the credit elasticity, the coefficients remain positive and significant across the entire size distribution, even for the top decile. A 1% increase in credit supply leads to a 0.145% increase in investment for the bottom decile and a 0.052% increase for the top decile.

Finally, we conduct the same robustness checks as in the credit regression. These include alternative size indicators—such as real turnover, lagged real total assets, and number of employees—winsorizing the shocks at the 1st and 99th percentiles, and controlling for size-dependent elasticity to credit demand shocks. The results, shown in panels (i) to (iv) of Figure 3 in Appendix B, are robust across all these different specifications.

## 3.3 Discussion

Overall, we find that there is a considerable average elasticity of credit and capital with respect to credit supply shocks and that these elasticities are only mildly decreasing and are significantly positive across the firm size distribution. In line with the two financial constraints definition put forward in section 2, this is suggestive evidence that the share of financially constrained firms may only be mildly declining and positive across the firm size distribution and suggestive of the existence of large constrained firms.

The literature has found evidence in support of our hypothesis. First, Bodnaruk et al. (2015) and Buehlmaier & Whited (2018), both using textual analysis, also document that some of the

U.S. publicly listed firms, which are among the largest firms in the world, also acknowledge they are financially constrained.

Second, according to Farre-Mensa & Ljungqvist (2016), financially constrained firms react more to credit supply shocks. If then, the mass of constrained firms would be considerably higher at the bottom of the size distribution than at the top, one would see the average elasticity to credit supply shocks decreasing on size. This is opposite to our findings, with the elasticity relatively constant over the size distribution.

Third, there is growing literature on the causal effect of financing constraints on firm level outcomes. Financially constrained firms, defined across a number of different measures, are found to have a higher elasticity of investment and employment with respect to shocks to the collateral value (Chaney et al., 2012), to financial shocks (Chodorow-Reich, 2014) and to monetary policy shocks (Greenwald et al., 2019; Ottonello & Winberry, 2020; Cloyne et al., 2023, for example). This is in line with our results and our hypothesis that credit constrained firms have a higher elasticity to credit supply shocks.

Despite this large amount of evidence, few studies explicitly consider the size-dependent elasticities and their aggregate implications. In the next section, we address this gap by using a structural heterogeneous firms model with financial frictions to analyze how these elasticities affect aggregate outcomes.

# 4 Model

In this section, we present a heterogeneous firms model with financial frictions to assess the aggregate implications of our empirically observed size-dependent elasticities from section 3. We build on Khan & Thomas (2013) and introduce ex-ante heterogeneity through a permanent productivity component. This addition can break up the strong correlation between size and financial constraints. Firms with lower permanent productivity will reach their optimal amount of capital and will be unconstrained from then on, while firms which draw a higher permanent component at birth may be constrained even when very large as they are still growing to reach their high potential. This gives the model the ability to better match the cross-sectional distribution of elasticities.

Note that we do find empirical evidence in support of a permanent productivity component. We use three different approaches, as described by Pugsley et al. (2021). Firstly, the standard deviation of employment across firms is high and increasing with age, implying large size differences early in the life cycle and a wide range of optimal firm sizes. Secondly, the autocorrelation of employment remains high throughout a firm's life cycle. These two descriptive results point towards the presence of permanent firm differences. Finally, we confirm the importance and differential incidence of ex-ante heterogeneity using the flexible statistical model developed by Pugsley et al. (2021). More details on the empirical analysis of the productivity process in our dataset can be found in Appendix D.

## 4.1 Households

The household sector of the model is deliberately simple. In particular, a representative household who owns the firms, chooses consumption, savings and labor supply according to the following maximization problem with Rogerson (1988) indivisible labor utility formulation:

$$V(k) = \max_{c,l,a'} \{\log(c) + \psi(1-l) + \beta \mathbb{E} V(a')\} \tag{3}$$

subject to:

$$a' + c = (1+r)a + \omega l + D, \tag{4}$$

where $c$ is consumption, $l$ is labor, $a$ is savings and $D$ are dividends. $\omega$ is the wage, $r$ is the real interest rate. The first-order conditions for the household problem are standard:

$$U_l(c,l) = \omega U_c(c,l) \tag{5}$$

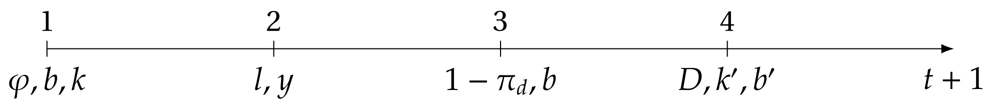
$$U_c(c,l) = \beta \mathbb{E}\left[(1+r')U_c(c',l')\right]. \tag{6}$$

And at the steady state, the first-order conditions are:

$$(1+r) = \frac{1}{\beta} \tag{7}$$

$$\omega = \psi c \tag{8}$$

Figure 4: Within period timing of incumbent firm




## 4.2 Production

The production sector features a continuum of firms, indexed by $i$. Firms are either classified as entrants or incumbents, as detailed below.

**Incumbents** An incumbent firm $i$ produces according to the following production function:

$$y_i = \varphi_i k_i^{\alpha} l_i^{\upsilon}, \quad \alpha + \upsilon < 1, \tag{9}$$

where $k$ and $l$ are capital and labor inputs and $\varphi$ denotes idiosyncratic productivity. Every firm's productivity comprises two components:

$$\ln \varphi_i = w_i + \theta_i, \tag{10}$$

where $w_i$ is an idiosyncratic *transitory* productivity shock, which follows an AR(1) process with persistence $\rho_w$ and variance of innovations $\sigma_\epsilon^2$. $\theta_i$ is the *permanent* productivity component, drawn at birth from a normal distribution with mean $\mu_\theta$ and variance $\sigma_\theta^2$

$$\theta_i \overset{\text{iid}}{\sim} \mathcal{N}\left(\mu_\theta, \sigma_\theta^2\right) \tag{11}$$

$$w_i' = \rho_w w_i + \varepsilon_i' \qquad \varepsilon_i \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \quad \rho_w < 1 \tag{12}$$

The firm's total profits before investment are revenue minus labor costs (in what follows we suppress $i$, the firm indicator, to ease on notation where possible):

$$\pi = y - \omega l, \tag{13}$$

where $\omega$ is the wage per unit of labor.

Figure 4 summarizes the within-period timing of the incumbent. The firm enters the period with predetermined levels of debt $b$, capital $k$, and immediately observes its idiosyncratic productivity $\varphi$ composed of a permanent and transitory component. Next, the firm's labor decision is a static choice that can be found through the firm's first order condition:

$$l(k,\varphi;\omega) = \Big(\frac{\upsilon\varphi}{\omega}k^{\alpha}\Big)^{\frac{1}{1-\upsilon}}. \tag{14}$$

After the production stage, the firm may suffer an exogenous exit shock. The shock happens with probability $\pi_d$. Consequently, the value of the firm after the production stage is given by

$$V^1(x,\varphi) = \pi_d x + (1-\pi_d)V^2(x,\varphi) \tag{15}$$

If the firm survives the exit shock, at the end of the period it chooses debt $b'$ and capital $k'$ to take to the next period and dividends to distribute this period $D$ to maximize its value

$$V^2(x,\varphi) = \max_{k',b',D}\Big[D + \mathbb{E}_{\varphi'|\varphi}\Lambda V^1(x',\varphi')\Big] \tag{16}$$

subject to:

$$D \equiv x + qb' - k' \geq 0 \tag{17}$$

$$b' \leq \xi x \tag{18}$$

$$x' \equiv x(k',b',\varphi') = y(l(k',z'),k',\varphi') - wl(k',\varphi') + (1-\delta)k' - b' \tag{19}$$

where $\xi$ is the financial parameter that captures the financial frictions in the economy, $x$ is the net worth with which the firm starts the period, given as the sum of profits plus the value of the non-depreciated capital minus the debt the firm has to pay back. $q$ is the price of the bonds firms issue, with $\frac{1}{q}-1$ equal to the equilibrium interest rate, $r$. $\Lambda$ is the firm discount factor. As the representative household is the owner of the firm, we assume $\Lambda = \beta$ in the steady state.

The firm faces two critical constraints according to (16). First, the firm cannot issue negative dividends or, equivalently, raise equity. Second, the firm is only able to borrow up to an exogenous fraction $\xi$ of its total cash on hand. We opt for a cash on hand collateral constraint following evidence from Lian & Ma (2021), which illustrates firms' debt contracts and financial constraints do not depend solely on assets, but also on the firm's value and cash flow. Our measure of cash on hand captures exactly these two sides, as it takes into account both the cash flow and the non-depreciated capital.[8]

**Entrants** Entry in this model is exogenous. We assume there is a fixed measure, $M_e$, of entrants equal to the mass of firms exiting after receiving a death shock. The entrants are assumed to enter with zero debt ($b_0 = 0$) and are log normally distributed over their initial capital $k_0$ with the mean anchored at a fraction of the mean of optimal capital levels. The choice of a log normal distribution is motivated by the right skewed distribution of entrants in the data. The initial productivity of each entrant, $\varphi_0$, follows the same process as the incumbents' productivity. Note that firm entry takes place at the end of a period, and entrants start operating in the next period, given their initial state, $(k_0, b_0, \varphi_0)$.

## 4.3 Firm level decisions

To characterize the firms' decisions we divide the firms into three groups, following Khan and Thomas (2013) and Jo & Senga (2019). This simplifies the solution of the model significantly.

1. **Unconstrained firms** Firms that can implement the optimal amount of capital and guarantee that in the future they will never be constrained again.
2. **Constrained firms, type 1** Firms that can implement the optimal amount of capital but not the minimum savings policy that guarantees they will never be constrained again in the future.
3. **Constrained firms, type 2** Firms that are constrained and cannot implement the optimal amount of capital nor the minimum savings policy.

For model details on the decisions of the firms in each group see Appendix C.1.

## 4.4 Solving and calibrating the model

**Solution Method** As outlined in Subsection 4.3, one can categorize firms into constrained, potentially constrained and unconstrained firms. The two cash-on-hand thresholds that define

[8]We focus on riskless debt contracts, which are fully collateralized, and so there is no dispersion on debt prices. See for example Cavalcanti et al. (2021) on how the price dimension affects firms' investment.

to which group a firm belongs are derived in Appendix C.1. One can then directly solve for the capital and bond policy function numerically.

To solve for aggregate quantities, we approximate the firm distribution over a fixed grid of net worth using the histogram method proposed by Young (2010).

The steady state solution is then given at the wage which is leading to a clearance of the goods market.[9] In the general equilibrium transitions the wage and interest rate adjusts to ensure that this is also true over the entire time horizon.

**Calibration** We calibrate the model in two steps, as is usual in the literature. First, we exogenously fix some of the parameters from the literature. Second, we then numerically calibrate some parameters to match critical moments in the data.

For most of the parameters, which are unrelated to distributions in the model, we follow Khan & Thomas (2013). The set of parameters chosen is documented in the upper part of Table C1 in Appendix C. The discount factor, $\beta$, is set to yield an average annual real interest rate of 4%. The production parameters, $\eta$ and $\alpha$, imply a labor share of 60% and capital share of 30%, respectively. Leisure preferences imply that households work one third of their available time.

The mean productivity levels for the permanent and transitory component, $\mu_\theta$ and $\mu_w$, are normalized such that when transforming them into a log-normal distribution, the expected productivity component equals one.[10] The rest of the parameters - collateral constraint $\xi$, standard deviation of permanent shock $\sigma_\theta$, persistence and standard deviation of the transitory shock $\rho_w$ and $\sigma_w$, exogenous probability of exit $\pi_d$, and the relative size and standard deviation of entrants $\mu_{ke}$ and $\sigma_{ke}$ - are calibrated using the simulated method of moments (SMM).

The values presented in the lower part of Table C1 in Appendix C minimize the distance between a set of empirical moments of the firm distribution. The moments chosen are commonly targeted in the literature to discipline the distribution of firms along the size dimension, and the life cycle of the firm. In particular, these moments condition the speed at which firms grow and reach their optimal size.[11] Additionally, we calibrate the credit and capital elasticities to a

[9] Market clearing interest rates at steady state are given by $1/\beta$ due to the household's first-order condition.

[10] Note that the mean of a log-normal distribution is affected not only by the location parameters but also the scale parameter. We adjust it accordingly, such that for any scale parameter, $\mu = 0$ yields an average productivity of 1, when transformed to a log-normal.

[11] See for example Midrigan & Xu (2014) or Khan & Thomas (2013). We use value-added for some of the mo-

Table 4: Calibrated model fit

| Moment | Model | Data |
|---|---|---|
| Size of 90th percentile / median | 7.716 | 9.070 |
| Gross leverage | 0.605 | 0.626 |
| Std. dev. of value added | 1.737 | 1.559 |
| 1-year autocorrelation of value-added | 0.971 | 0.924 |
| 5-year autocorrelation of value-added | 0.840 | 0.818 |
| Elasticity of debt to supply | 0.346 | 0.372 |
| Elasticity of capital to supply | 0.136 | 0.126 |
| Relative elasticity of capital: 90th percentile / 20th percentile | 0.370 | 0.359 |

*Note.* This table presents the moments used for our baseline calibration of the model. We use the identity matrix when computing the squared sum of residuals between model and data moments.

credit supply shock in the model to match the result in Tables 2 and 3 respectively. Lastly, we target the ratio of the capital elasticity in the top decile of firm size, relative to the bottom 20%, to discipline the evolution of the capital elasticity across the size distribution.

Table 4 compares the targeted moments in the data and in the model, demonstrating a good fit with an average error of approximately 5%.

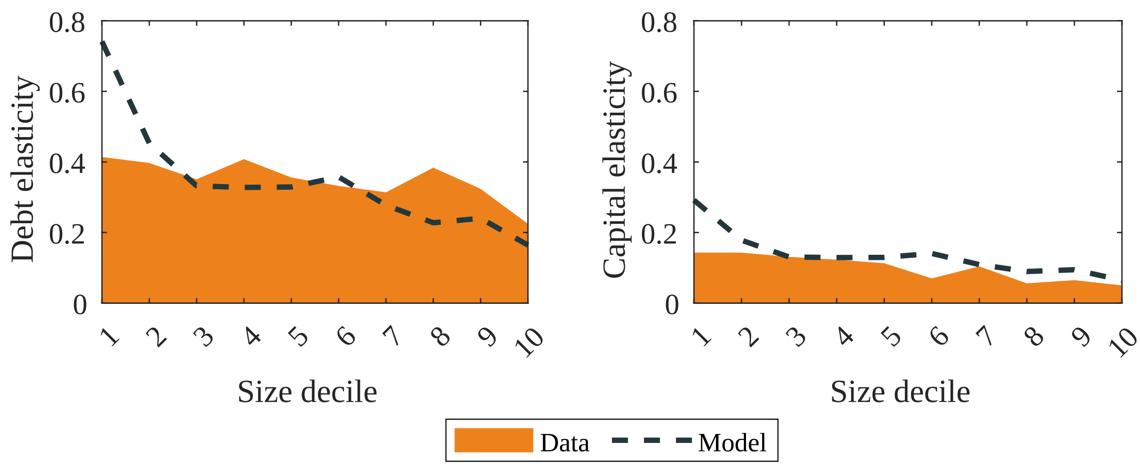
**Non-targeted moments** The model also generates a joint size-elasticity firm distribution, for both credit and capital, that is in line with the data as illustrated in Figure 5. On the left panel the credit elasticity in the data (orange area) and in the model (dashed black line) are depicted, while on the right panel it is a similar figure for the capital elasticity. As can be seen from both figures, both the credit and capital elasticity in the data and in the model follow closely the empirical counterparts.

Notice that, in the model, the credit elasticity of constrained firms to a credit supply shock is equal to one, while unconstrained firms have an elasticity of zero. This means, the model generates small unconstrained firms as well as large constrained firms, which is what allows the model to closely approximate the untargeted elasticity deciles of the empirical distribution quite well, as depicted on the figure. It still slightly overestimates the elasticity of firms at the bottom of the distribution, but the remaining deciles are close to the data counterpart.

The distribution generated by the model is explained by the fact that some larger firms are

ments as it is the closest counterpart to revenues in the model given that we abstract from intermediate goods.

Figure 5: Credit elasticity of firms across the distribution.




*Note.* The figure plots the elasticity of debt and capital with respect to credit supply. Left panel, credit elasticity with empirical results from the left panel of Figure 2. Right panel, capital elasticity with empirical results from the left panel of Figure 3.

still growing to reach their steady state capital and are still constrained. At the same time, the model accounts for a larger share of small firms that are born at or close to their steady state level of capital.[12]

# 5 Aggregate Implications

In this section we assess the implications of accounting for large constrained firms when faced with an aggregate financial or a TFP shock, respectively.

We proceed to assess the aggregate implications of having constrained firms across the entire firm size distribution. We start with the financial shock, assuming a drop in the maximum borrowing capacity of 50%.[13] The shock follows the following process:

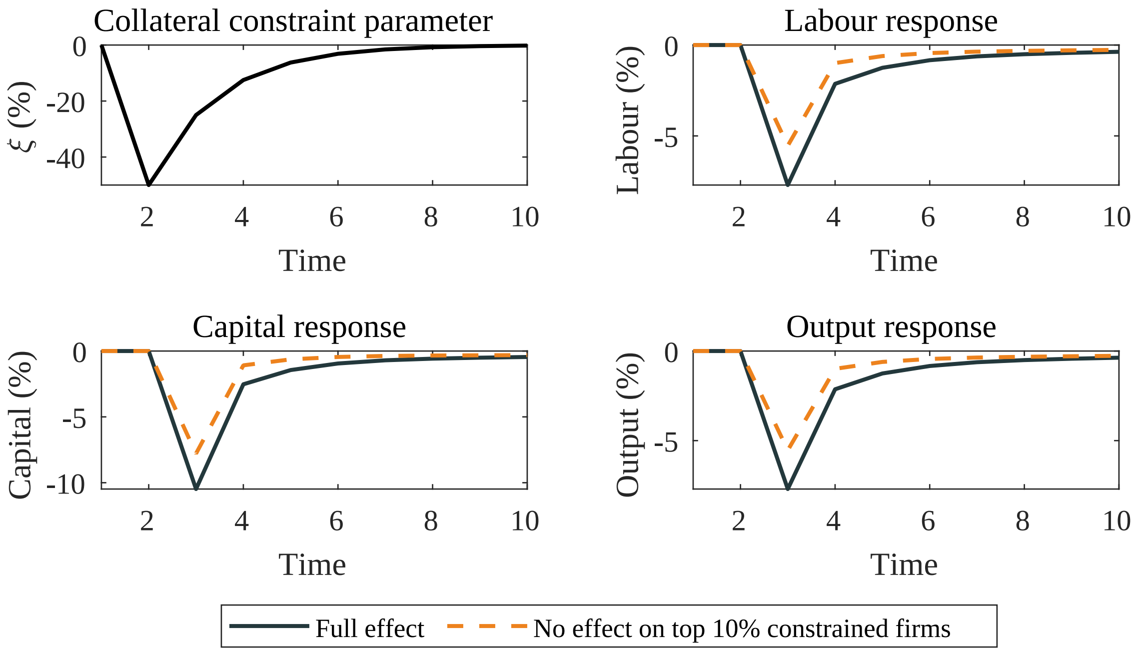
$$\xi_t = (1 - \rho_\xi)\xi_{ss} + \rho_\xi \xi_{t-1}, \tag{20}$$

with $\rho_\xi$ set to 0.5, so that the shock dissipates after 10 periods. Given the sudden and transitory nature of the financial shock, we assume wages to be fixed at the steady state level over the

[12]Figure C1 in Appendix C offers a slightly different perspective, plotting the density distribution of constrained and unconstrained firms. It is possible to observe that the distributions of constrained and unconstrained overlap.

[13]Khan & Thomas (2013) simulate an 88 percentage point drop in $\xi$. However, in their calibration the initial level of $\xi$ is 1.38. In our calibration $\xi$ is 0.68, hence a 50% drop equals a 34 percentage point drop in maximum borrowing allowances.

Figure 6: IRFs to a financial shock.




*Note.* Lines indicate the partial equilibrium response to a reduction of $\xi$ in the upper left panel, with wages and debt prices fixed at their steady state level.

transition.[14]

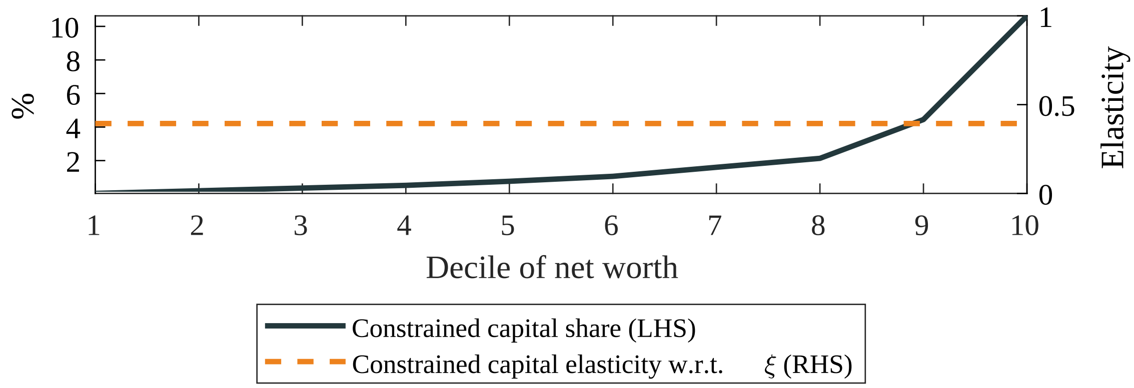
**Financial shock and large constrained firms** Figure 6 shows the responses to the credit shock depicted in the upper left panel. Since the firm's capital stock is predetermined, there is no direct impact in period $t = 2$, when the financial shock hits. However, the lower maximum borrowing capacity affects constrained firms in their investment decision, whilst unconstrained firms remain unaffected by the shock as their borrowing constraint is not binding.

The resulting aggregate effect of constrained firms reducing their investment depends heavily on the distribution of these constrained firms along the firm size distribution. The dark blue line in Figure 6 illustrates the overall effect of a financial shock on labor, capital and output. Output drops by 7.7% in response to a 50% decrease in the collateral constraint parameter. The dashed orange line shows the aggregate responses in the scenario that the 10% largest constrained firms behave as unconstrained firms.[15] The overall effects are reduced by approxi-

[14]General equilibrium results for this exercise lead to the same qualitative and quantitative conclusions, but we prefer the partial equilibrium analysis to isolate the effect coming from the differences in the distribution of constrained firms. The GE IRF to a financial shock is depicted in Figure C2 in Appendix C.

[15]The mass of constrained firms is 35% of total firms. As such, the top 10% of constrained firms are only 3.5%

Figure 7: Elasticity with respect to $\xi$ and share of capital in constrained firms




*Note.* The left y-axis measures the share of capital whilst the right y-axis measures the elasticity of capital with respect to the borrowing constraint parameter $\xi$. Both of these statistics are shown for constrained firms. The x-axis tracks the decile of net worth in the distribution of all firms

mately 30%, with output only dropping 5.5% in this scenario. This illustrates that the top 10% of constrained firms account for close to one-third of the overall decrease in output in response to a financial shock.

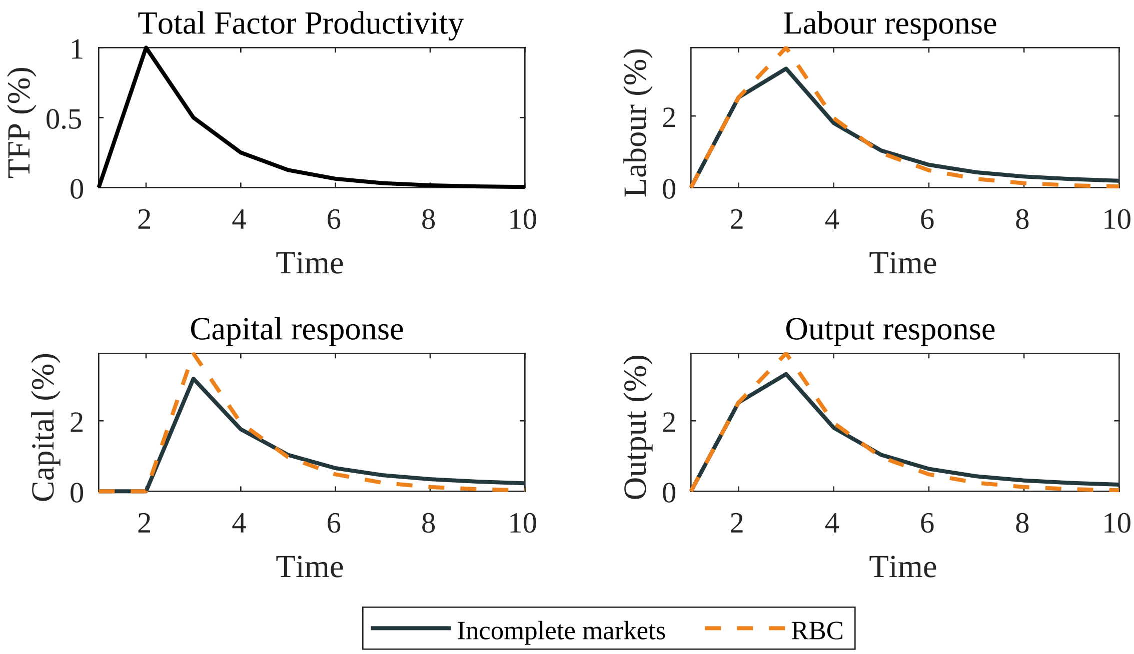
The quantitative magnitude of the effect clearly depends on the elasticity of capital with respect to the financial shock. While we calibrate the model to the average elasticity in Table 3, this is the lowest average elasticity across all our different empirical specifications, with the range of average elasticities spanning from 0.126 in the benchmark scenario, to 0.158 for the specification with winsorized shocks and controls. The model generates an average capital elasticity to supply shocks of 0.135, on the lower end of the empirically estimated elasticities.

Additionally, we calibrate the model to match the 90th to 20th percentile ratio from Figure 3. The model predicts a capital elasticity on the top 10% of the size distribution of 0.078. The highest empirically estimated elasticity for the top 10% of firms is 0.095, which would imply an even larger aggregate response to the shock.

**Key Mechanism** Figure 7 illustrates the key mechanism at play. The dashed orange line presents the elasticity of constrained firms with respect to $\xi$, which is independent of size. For unconstrained firms, as already pointed out, the elasticity is zero. The dark blue line is indicating the share of productive capital in constrained firms by net worth decile. Despite the share of constrained firms at the top of the size distribution being smaller than at the bottom of the dis-

of total firms in the economy.

Figure 8: IRFs to a TFP shock.




*Note.* Lines indicate the partial equilibrium response to a shock to $A$ in the upper left panel, with wages fixed at their steady state level.

tribution the percentage of productive capital in this large constrained firms is much higher. This together with the fact that the elasticity of constrained firms to the shock is independent of size, explains why large constrained firms account for around one-third of the overall response of output to the financial shock.

The mechanism holds even when we consider the general equilibrium transition. In this case, given the drop in capital in response to the financial shock, wage decreases, which causes labor and consequently output to not fall by as much as in the partial equilibrium scenario. Still, in the general equilibrium case the large constrained firms account for more than one third of the overall decrease in output. The results are presented in Figure C2 in Appendix C

**TFP shock and large constrained firms** Additionally, in Figure 8 we consider an unexpected and temporary 1% increase in total factor productivity (TFP). The shock follows the following process:

$$A_t = (1 - \rho_A)A_{ss} + \rho_A A_{t-1}, \tag{21}$$

with $\rho_A$ set to 0.5, so that the shock dissipates after 10 periods. We compare the response of the incomplete markets model to the case when $\xi$ tends to infinity, with all remaining parameters staying the same, in which case the model resembles a complete markets RBC model.[16]

In a direct response to the shock, firms employ more labor for any predetermined level of capital. In the second period, given capital is predetermined, the output response is driven by the labor response and there is no difference in the output response across the two models. In the third period, firms start to adjust capital and differences between the two models start to show. With financially constrained firms not being able to optimally adjust capital in response to the shock, the response in the incomplete markets case is muted and smaller than in the standard RBC.[17] Again, the presence of large constrained firms is key, as those firms account for a large fraction of productive capital in the economy and of the overall capital response. If constrained firms were all small and would own a small fraction of productive capital in the economy, the capital response across the two models would be very similar.

In general equilibrium the results of the heterogeneous firms model and RBC coincide, as depicted in Figure C3 in Appendix C. This is due to the dynamics of the real interest rate. For a detailed discussion on how to make the dynamics of real interest rate consistent with empirical evidence, please see Winberry (2021).

**Policy scenario** Lastly, we explore a policy that could limit the negative effects of the financial shock. We assume the government imposes a one-off lump sum tax on households, which amounts to 1% of steady state consumption. The government then distributes the tax revenues as a lump sum subsidy across firms on the different size bins at the impact of the financial shock. We compare four different policies: 1) subsidizing uniformly all firms in the economy; 2) subsidizing only the smallest 10% firms; 3) subsidizing only the largest 10% firms; and 4) the optimal policy, which reduces output losses the most.

Table 5 compares the temporary and cumulative output losses in partial equilibrium and general equilibrium following the financial shock under the different policies and the bench-

[16]Notice that while there is still dispersion in the permanent and transitory productivity component, all firms produce at their optimum and MPKs are equalized across firms, effectively approximating a representative firm model.

[17]Note, the effect could flip if the borrowing constraint was cyclical, which would cause the elasticity of constrained firms to become larger than that of unconstrained ones.

Table 5: Impact and cumulative output loss for different size-based policies

| | Partial Equilibrium | | General Equilibrium | | |
|---|---|---|---|---|---|
| | $\frac{Y_3}{Y_{SS}}$ | $\frac{\sum_{t=1}^{10}\beta^{t-1}Y_t}{Y_{SS}}$ | $\frac{Y_2}{Y_{SS}}$ | $\frac{Y_3}{Y_{SS}}$ | $\frac{\sum_{t=1}^{10}\beta^{t-1}Y_t}{Y_{SS}}$ |
| Benchmark | -7.70 | -12.17 | -4.64 | -4.44 | -7.60 |
| Top 10% | -7.42 | -11.76 | -4.39 | -4.09 | -7.11 |
| Uniform | -7.35 | -10.42 | -4.43 | -4.05 | -6.73 |
| Bottom 10% | -7.22 | -7.90 | -4.50 | -3.97 | -5.97 |
| Optimal | -7.22 | -7.90 | - | - | - |

*Note.* This table presents the on impact output loss and cumulative output loss in the benchmark case (no policy) and the four different policies we implement. Notice in the Partial Equilibrium in period 2, due to prices being fixed, output does not react. For the General Equilibrium case we do not compute the optimal policy.

mark case without any subsidy. Despite the fact that large constrained firms drive a sizeable portion of output losses on impact, the table illustrates that subsidizing small firms results in larger cumulative gains than subsidizing large firms. This is evident from the smaller output drop in period three and the cumulative loss when the government subsidizes exclusively the smallest 10% of firms. This holds true in both partial and general equilibrium. However, in period two of the general equilibrium case, the order reverses due to price adjustments taking effect at that stage. Nonetheless, the overall cumulative output loss remains lower when targeting the bottom 10%, as shown in the final column of the table. Finally, when we compute the optimal policy for the partial equilibrium case we find that the prescription is again to finance only the smallest 10% of firms.[18]

The average capital elasticity by size decile depicted in Figure 5 helps to rationalize the result on impact. As the average capital elasticity at the bottom of the size distribution is around 0.3 while at the top is 0.1, the government will get an additional investment of 0.3 out of each additional unit of subsidy given to the smaller firms, while this figure would only be 0.1 of additional investment for the top 10% of firms. The higher elasticity at the bottom of the size distribution is reflective of the strongest prevalence of financial constraints at the bottom of the size distribution than at the top.[19]

[18]We do not compute the general equilibrium optimal policy due to the high computational costs involved in solving for the general equilibrium transition.

[19]If the government can subsidize directly constrained firms, that would be optimal as those firms are the most affected by the financial shock. In case the government cannot directly observe which firms are financially constrained, it should allocate the subsidies to the bottom of the size distribution where financial constraints are more prevalent.

Additionally, subsidizing the smallest firms results in a 33% smaller cumulative output loss. As smaller firms have a higher growth potential than larger ones, the economy recovers faster to the steady state.

## 6 Conclusion

This paper documents the importance of matching size-dependent credit and capital elasticities to credit supply shocks. We find both these elasticities to be only mildly decreasing with size, which is suggestive of the existence of large financially constrained firms. We subsequently analyze the importance of matching this fact in a quantitative financial frictions model with heterogeneous firms.

In order to do so, we build a standard firm dynamics model, with a richer productivity, which is supported by empirical evidence from our study and previous research. We demonstrate that adding a permanent component to the productivity process helps the model generate a joint size-elasticity firm distribution in line with the data, breaking the typically strong correlation between financial constraints and size and generating a sizeable mass of small unconstrained and large constrained firms. The existence of large constrained firms consequently drives up the share of productive capital in this type of firms. This, together with the fact that constrained firms have a distinct elasticity from unconstrained firms, has significant implications for aggregate responses to aggregate financial and TFP shocks. In particular, the effects of a financial shock are strongly influenced by the presence of large constrained firms, with the largest 10% of constrained firms explaining close to one-third of the output drop in response to a financial shock. In response to a positive aggregate TFP shock, we find the output response to be 15% smaller than in a standard RBC model, due to the more muted response of constrained firms.

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