Online Appendix

Financial Constraints and Firm Size: Micro-Evidence and Aggregate Implications

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A Data Description

A.1 Definition of Variables in the Central Credit Responsibility Database (Central de Responsabilidades de Crédito)

Identifier *(tina)*: Anonymized tax identification number.

Global Credit *(valor global)*: The sum of effective credit and potential credit, representing the total available credit that a firm accesses.

Effective Credit *(valor efectivo)*: Credit effectively used in a regular situation, i.e., without payment delays as defined in the respective contract. Examples of effective responsibilities are:

- Loans for the acquisition of financial instruments (shares, bonds, etc.);
- Discount and other credits secured by effects;
- Overdrafts on bank accounts;
- Leasing and factoring;
- Used amounts of credit cards.

Potential Credit *(valor potencial)*: Irrevocable commitments of the participating entities. Banco de Portugal requires all credit-granting institutions to report to the CCR their outstanding loan exposure by instrument of all irrevocable credit obligations. Examples of potential responsibilities are:

- Unused amounts of credit cards;
- Lines of credit;
- Guarantees provided by participating entities;
- Guarantees and guarantees given in favor of the participating entities;
- Any other credit facilities likely to be converted into effective debts.

Overdue Credit *(valor vencido)*: All outstanding credit exposures recorded as non-performing (including overdue, written off, renegotiated credit, overdue credit in litigation, and written off credit in litigation) are aggregated to calculate overdue credits. This includes principal, interest and related fees.

Short-term Credit *(valor curto)*: Short-term credit is calculated using two different definitions. First, short-term credit is defined based on the term-to-maturity as agreed in the credit contract, denoted by valor_curto_o. Specifically, short-term credit has original maturity of equal to or less than one year. Before 2009, the CCR dataset did not streamline credit exposure based on the maturity structure. Therefore, for data before 2009, short-term credit is defined as the aggregation of commercial credit, discount funding, and other short-term funding, which are inherently short-term. Second, short-term credit is defined based on residual maturity, i.e. the remaining time until the expiration or the repayment of the instrument, denoted by valor_curto_r. Specifically, it refers to credit with residual maturity of equal to or less than one year. This variable is only available from 2009 onwards. Potential credit is excluded for both calculations.

Long-term Credit *(valor longo)*: Similar to short-term credit, long-term credit is defined by original and residual maturities. More precisely, long-term credit is credit with an original or residual maturity of more than one year, denoted by valor_longo_o and valor_longo_r, respectively. Long-term credit defined on an original maturity basis (valor_longo_o) for the data before 2009 is the aggregation of total credit excluding commercial credit (type 1), discount funding (type 2), and other short-term funding (type 3). Potential credit is excluded for both calculations.

A.2 Descriptive Statistics

Table A1: Descriptive statistics of Portuguese firms between 2006 and 2017

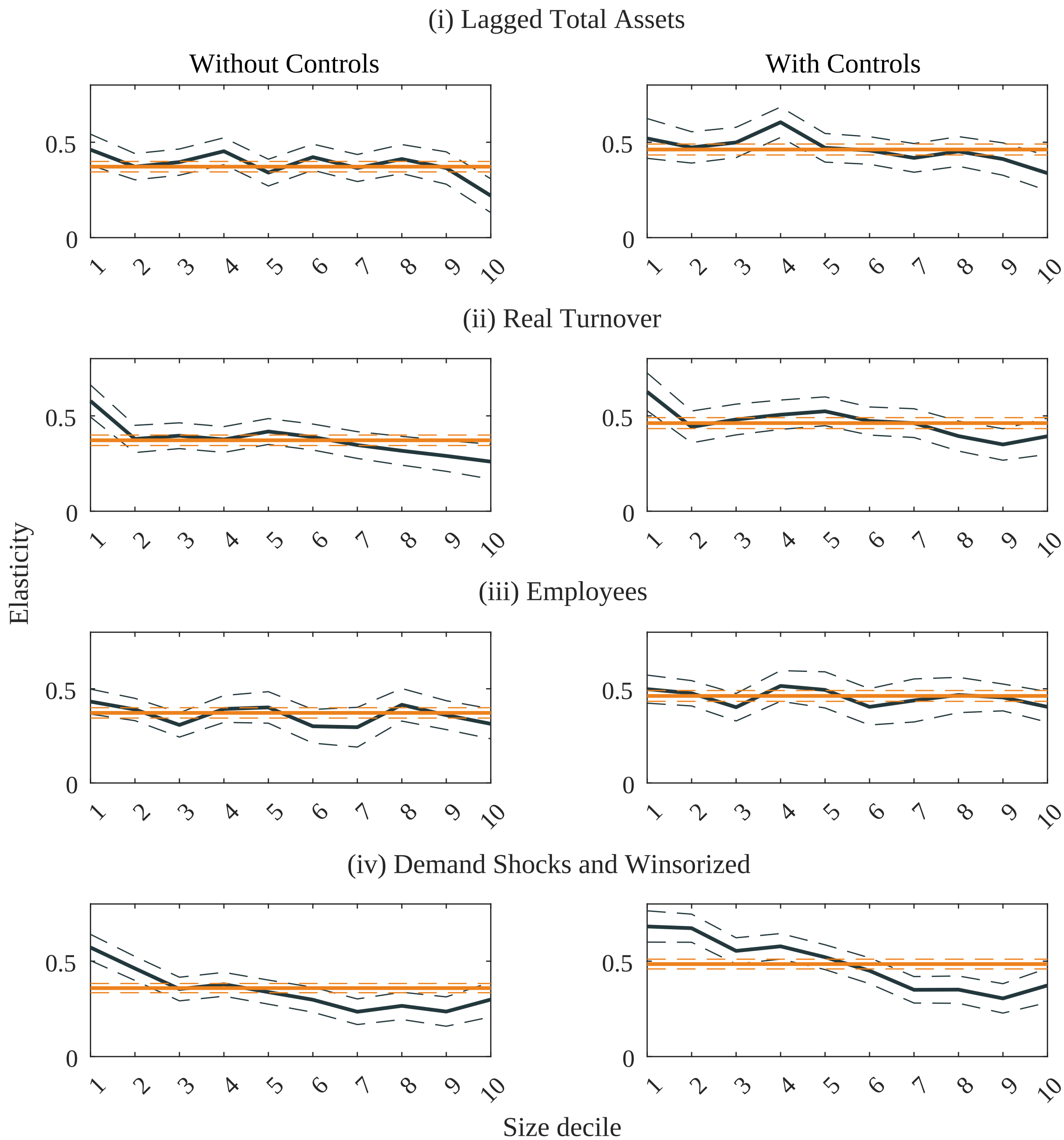
Variable	Mean	Median	Std. Dev.	Size group median			
				90th	90th-99th	99-99.5th	>99.5th
Total Assets	3.15	0.28	85.10	0.25	5.06	42.71	135.70
Turnover	1.86	0.23	33.59	0.21	3.25	19.93	27.94
Potential credit	0.19	0.03	4.56	0.03	0.14	0.95	2.95
Effective credit	0.53	0.04	5.96	0.04	1.15	6.93	126.73
Leverage	0.28	0.20	0.38	0.20	0.24	0.17	0.08
Liquidity ratio	0.14	0.06	0.19	0.06	0.02	0.01	0.01
Age	15.01	12.00	12.26	12.00	21.00	23.00	21.50
Employees	14.47	4.00	130.58	4.00	25.00	95.00	98.00
# Banks	2.45	2.00	1.89	2.00	4.00	4.00	5.00

Note. Total assets, turnover, potential credit and effective credit are measured in 2010 Euro Millions.

B Empirical results: Robustness

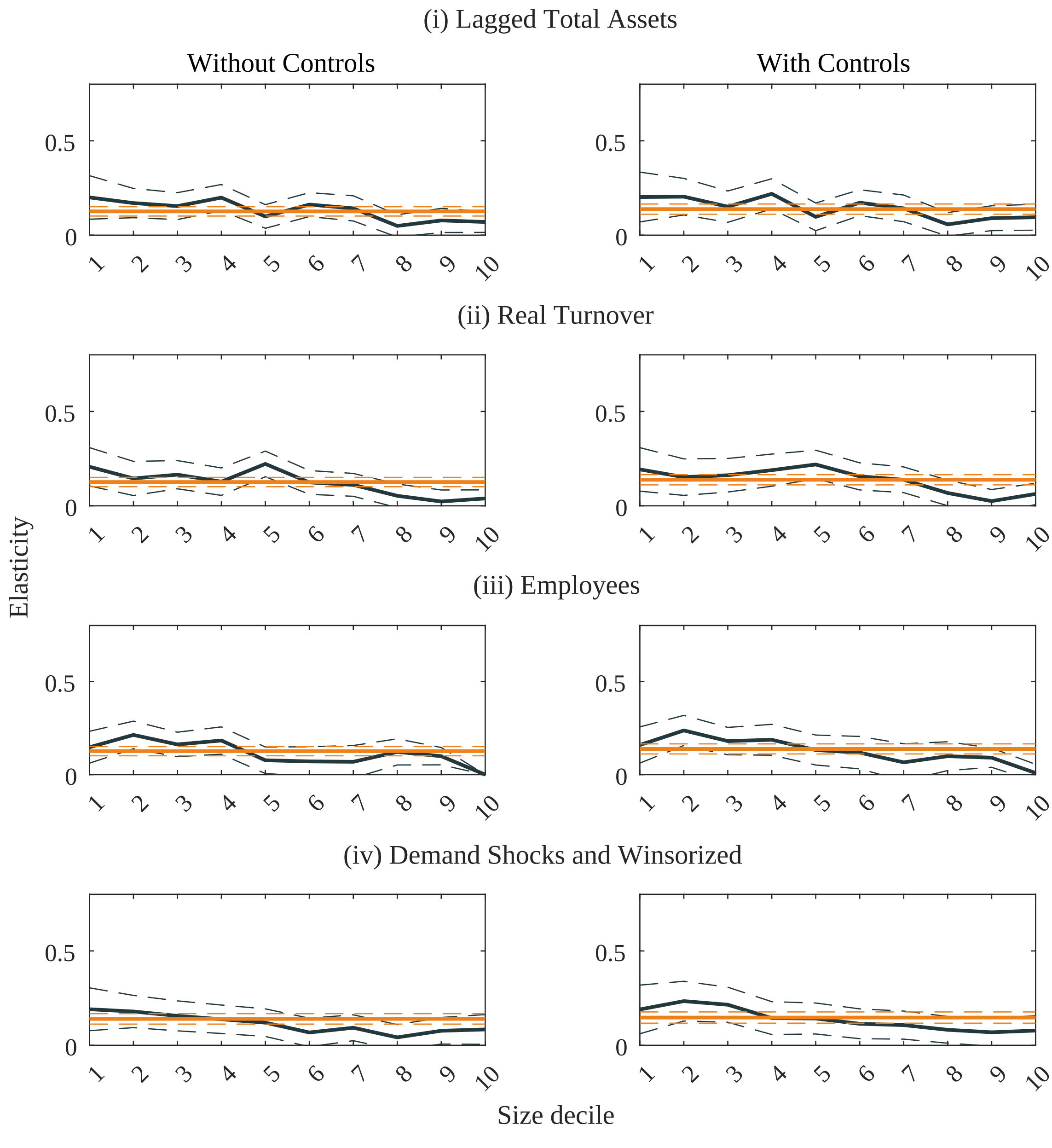
In this section we report a number of robustness exercises with respect to our main empirical specification in Section 3.2.

Figure B1: Average and size dependent elasticity of credit to bank supply shocks



Note. The panel shows different robustness exercises for the size-dependent elasticities. Row (i) shows results when we use lagged total assets as the size variable, row (ii) when use real turnover, row (iii) when we use employees and row (iv) shows results when we also include credit demand shocks and winsorize the shocks at the 1 % level in order to control for outliers. The dependent variable is credit growth. The first column shows results without extra controls, whilst the second column shows results with extra bank and firm controls

Figure B2: Average and size dependent elasticity of capital to credit supply shocks



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C Quantitative model

C.1 Firm level decisions

Unconstrained Firms This group of firms can implement both the optimal amount of capital and the minimum savings policy that guarantees these firms will never be constrained in the future again. Given the absence of adjustment costs and the stochastic process for φ the optimal amount of capital is the solution to:

$$\max_{k'} \quad -k' + \beta \mathbb{E}_{\varphi'|\varphi} \left[(\pi(k', \varphi') + (1-\delta)k' \right] \tag{1}$$

So the optimal amount of capital solves the following equation

$$\beta \mathbb{E}_{\varphi'|\varphi} \left[\frac{\partial \pi}{\partial k'}(k', \varphi') \right] = 1 + \beta\delta - \beta \tag{2}$$

which is when the expected marginal productivity of capital is equal to the marginal cost of an extra unit. The minimum savings policy these firms implement guarantees they will never be constrained again. It is given by

$$B^*(\varphi_i) = \min_{\varphi_j} \tilde{B}(k^*(\varphi_i), \varphi_j) \tag{3}$$

where $\tilde{B}(k^*(\varphi_i), \varphi_j)$ is the minimum savings that guarantees that going from state φ_i to φ_j the firm is still able to implement the optimal amount of capital. It is given by

$$\begin{aligned} \tilde{B}(k^*(\varphi_i), \varphi_j) = & \pi(k^*(\varphi_i), \varphi_j) + (1-\delta)k^*(\varphi_i) - k'^*(\varphi_j) + \\ & q \min\left\{ B^*(\varphi_j), \xi \left(\pi(k^*(\varphi_i), \varphi_j) + (1-\delta)k^*(\varphi_i) - \tilde{B}(k^*(\varphi_i), \varphi_j) \right) \right\} \end{aligned}$$

Given the optimal amount of capital and the minimum savings policy, the dividends distributed by the unconstrained firms are given by

$$D = x - k^* + qB^* \tag{4}$$

From the dividend constraint $D \geq 0$ we can extract the minimum threshold for cash-on-hand that guarantees the firm is not constrained

$$\tilde{x} = k^* - qB^* \tag{5}$$

and the firms is constrained if $x \leq \tilde{x}$.

Constrained Firms: Type 1 These firms can implement the optimal amount of capital, k^*, but not the optimal savings policy and are therefore partially constrained. As they may still be constrained in future states, they value internal financing more than households value dividends. As a result, for this type of firms, $D = 0$. The amount of debt is given by

$$b' = \frac{(k^* - x)}{q} \tag{6}$$

A firm is type 1 if it can adopt the above amount of debt and capital and at the same time guaranteeing that it does not default in the next period.

Constrained Firms: Type 2 Strictly constrained firms can not implement the optimal amount of capital. Those firms utilize all their borrowing capacity as their marginal value of net worth is greater than unity. Hence, their savings policy is simply

$$b' = \xi x, \tag{7}$$

and their maximum possible investment is consequently

$$k' = x + q\xi x < k^*, \tag{8}$$

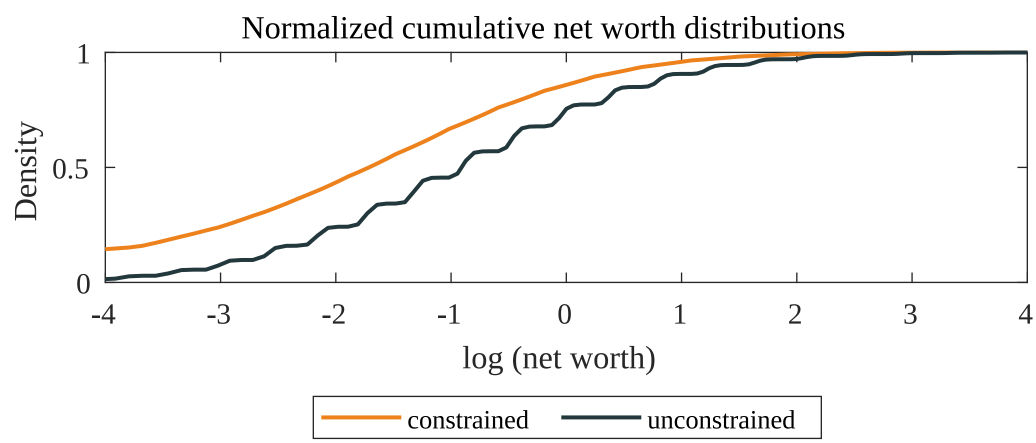
which is strictly smaller than their optimal level of capital k^*.

Table C1: Parameter values benchmark calibration

Parameter	Description	Value	Source
		Preset	
β	Discount factor	0.96	K&T (2013)
α	Returns on capital	0.27	K&T (2013)
η	Returns on labor	0.60	K&T (2013)
δ	Depreciation rate	0.065	K&T (2013)
ψ	Labour preference	2.15	K&T (2013)
μ_θ	Average: permanent productivity	0	Normalized
μ_w	Average: transitory shock	0	Normalized
		Numerically calibrated	
σ_θ	Std. dev.: permanent productivity	0.192	Calibrated
ρ_w	Persistence of transitory shock	0.009	Calibrated
σ_w	Std. dev: transitory shock	0.088	Calibrated
μ_{ke}	Relative size of entrants	0.361	Calibrated
σ_{ke}	Standard deviation of entrants	5.240	Calibrated
ξ	Collateral constraint	0.676	Calibrated
π_d	Exogenous probability of exit	0.049	Calibrated

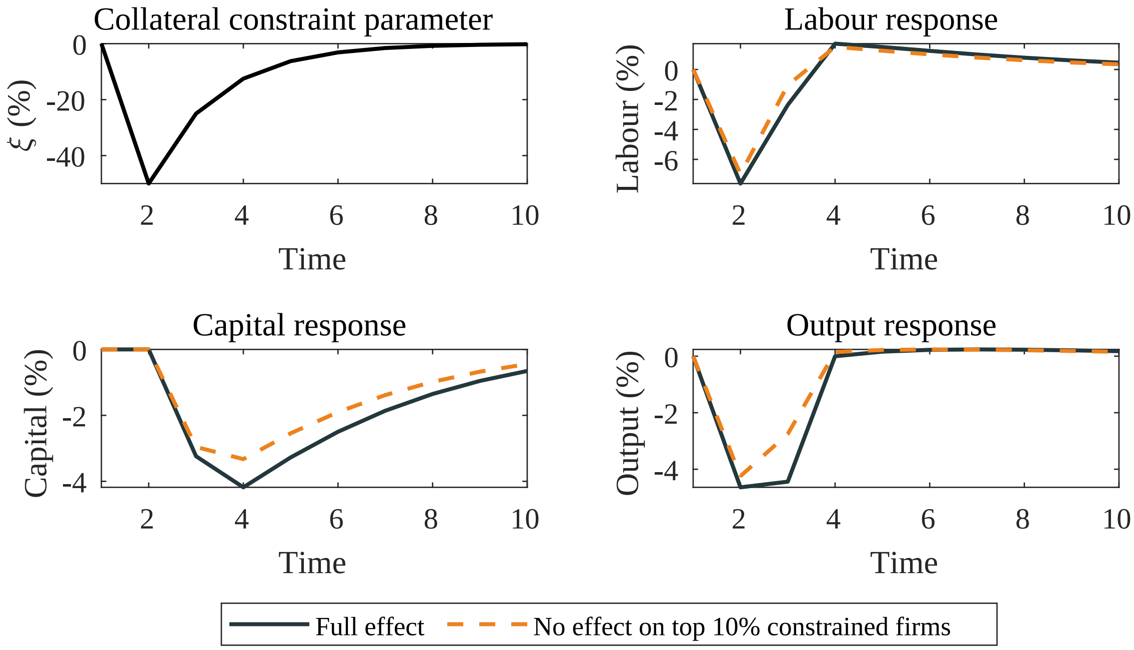
Note. K&T (2013) is short for Khan & Thomas (2013).

Figure C1: Conditional distributions of log of total assets implied by the model



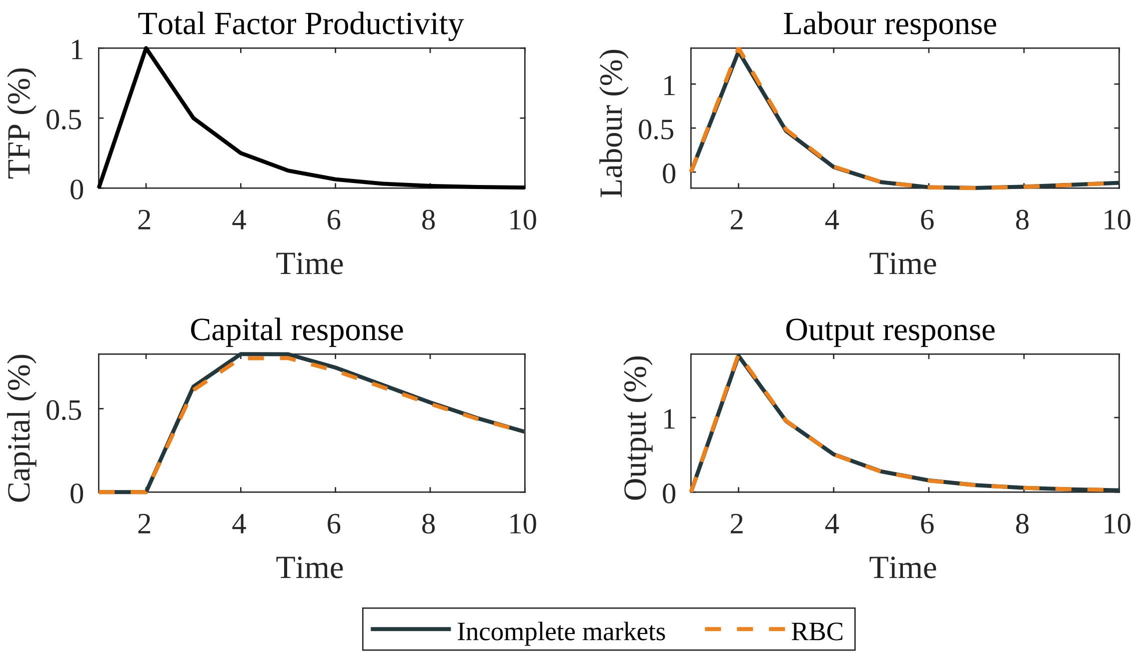
Note. The figure plots the density of firms that are constrained and unconstrained as a function of net worth x. For easier comparison, the densities are normalized so that their sum is equal to one in both cases.

Figure C2: General equilibrium IRFs to a financial shock.



Note. Lines indicate the general equilibrium response to a shock to ξ in the upper left panel.

Figure C3: General equilibrium IRFs to a TFP shock.



Note. Lines indicate the general equilibrium response to a TFP shock in the upper left panel.

D Firm potential

One factor we use to enable our model to match the size elasticity profile are heterogeneous ex-ante conditions for firms, such as firm potential. Small firms may be unconstrained as they already have reached their potential - i.e. optimal size - while large firms may still be growing and are still constrained. Equally, heterogeneous potential creates a dispersion of unconstrained firms across the entire firm size distribution. This can explain the declining but positive elasticity profile across the size distribution. Accordingly, this appendix investigates whether such ex-ante heterogeneity exists in Portugal.

Autocorrelation and standard deviation If there is heterogeneous firm potential, we should observe a relatively constant profile of standard deviation of employment over age and relatively high autocorrelation between employment over ages. Figure D1 plots these two statistics in our data, with panel (a) depicting the standard deviation and panel (b) depicting the autocorrelation profile.[1] First, the standard deviation is increasing over ages - we interpret that as evidence that there is no uniform size for firms. Second, the autocorrelation is relatively high between age groups, suggesting that initial conditions are also very important for firm growth. We interpret this affirmative evidence in favor of the importance of ex-ante conditions, in line with evidence presented by Pugsley et al. (2021).

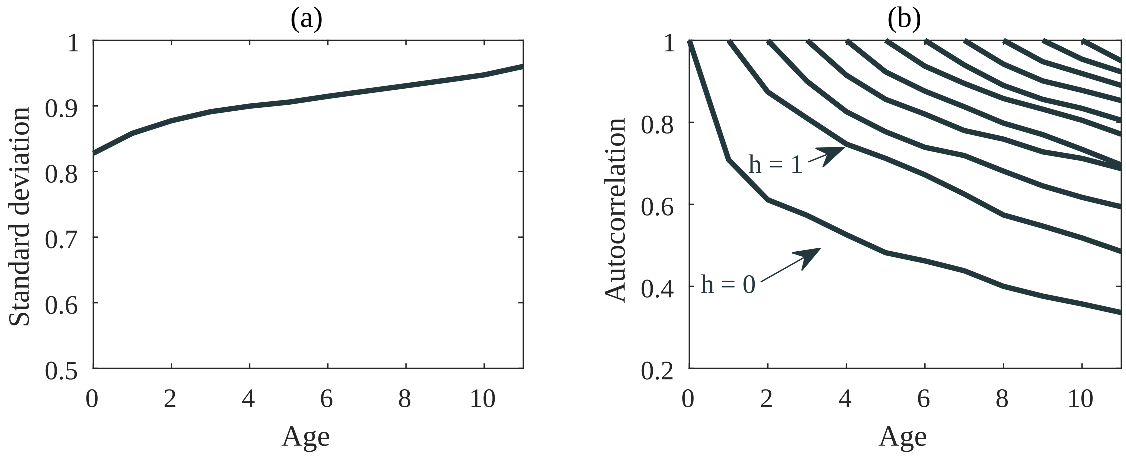
Statistical model To gain a more structural understanding beyond descriptive statistics of the importance of ex-ante and ex-post heterogeneity for the life-cycle of firms, we again follow Pugsley et al. (2021) and adopt their statistical model. This model uses the information provided by the autocovariance structure of log employment to capture the importance of both types of heterogeneity.

Consider the following decomposition for employment n by firm i at age a:

$$\underbrace{\ln n_{i,a}}_{\text{log employment}} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante component}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post component}}, \tag{9}$$

[1] To prevent differences across sectors and business cycle conditions from explaining the majority of the standard deviation and autocorrelation, we first control for sector and year fixed effects and then use the residuals of log employment.

Figure D1: Standard deviation and autocorrelation of log employment by age.



Note. The left panel presents the standard deviation of log employment by age, after controlling for sector and year fixed effects. The right panel presents the autocorrelation of log employment between ages a and $h \leq a$. Across lines h changes, while a changes along the lines

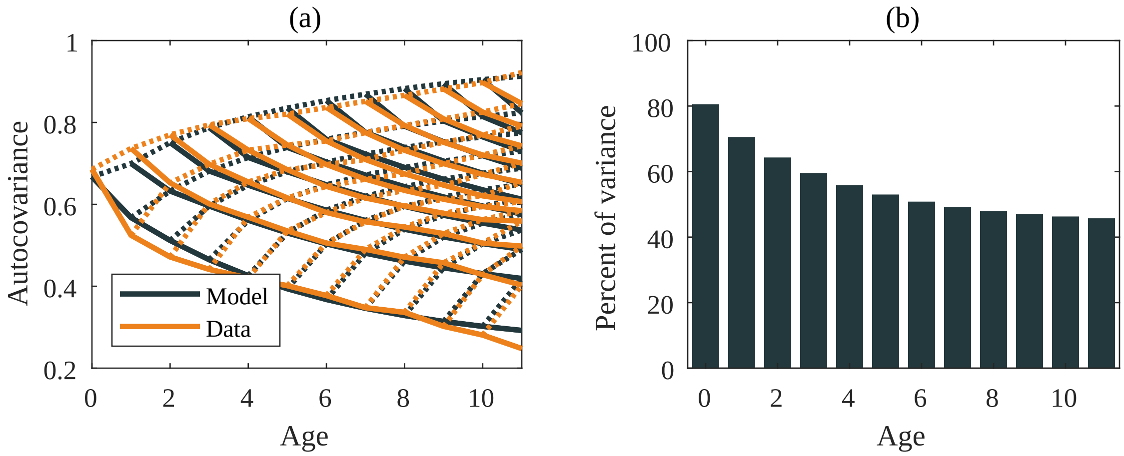
where

$$\begin{aligned} u_{i,a} =& \ \rho_u u_{i,a-1} + \theta_i, && u_{i,-1} \sim iid\left(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}\right), && \theta_i \sim iid\left(\mu_\theta, \sigma^2_\theta\right), && \left|\rho_u\right| \leq 1 \\ v_{i,a} =& \ \rho_v v_{i,a-1}, && v_{i,-1} \sim iid\left(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}\right), && && \left|\rho_v\right| \leq 1 \\ w_{i,a} =& \ \rho_w w_{i,a-1} + \varepsilon_{i,a}, && w_{i,-1} = 0, && \varepsilon_{i,a} \sim iid\left(0, \sigma^2_\varepsilon\right), && \left|\rho_w\right| \leq 1 \\ z_{i,a} \sim& \ iid\left(0, \sigma^2_z\right) \end{aligned}$$

In this employment process, the terms $u_{i,a}$ and $v_{i,a}$ capture the ex-ante profile while $w_{i,a}$ and $z_{i,a}$ capture the ex-post one. The ex-ante component is determined by three shocks that are drawn just prior to the birth year, at $a = -1$. The shocks $v_{i,-1}$ and $u_{i,-1}$ represent the initial conditions of the firm, which allow for rich heterogeneity even at birth. θ_i is the permanent component, which will accumulate over the life-cycle at speed ρ_u. In particular, with $\rho_u < 1$, the long-run steady state level of employment will be given by $\frac{\theta_i}{1-\rho_u}$. Further, this specification allows for rich heterogeneity not only in terms of optimal size of the firms, depending on the distribution of θ_i, but also in terms of the speed at which firms reach the steady state. As firms start at different points depending on $u_{i,-1}$ and $v_{i,-1}$ and each shock has its own persistence parameter, the path from initial to steady state employment will highly differ across firms.

The ex-post component is formed of two different shocks, one i.i.d. shock with expected value of zero, and a persistent one that follows an AR(1) process with i.i.d. innovations $\epsilon_{i,a}$ and persistence ρ_w. To abstract the ex-post component from affecting the ex-ante one, we set the

Figure D2: Model fit and ex-ante variance contribution



Note. Panel (a) plots both the autocovariance structure of residual employment in the calibrated statistical model (blue) and in the data (orange). Panel (b) presents the percent of variance explained by ex-ante factors as a function of age according to the statistical model.

initial conditions of the persistent shock to $w_{i,-1} = 0$.

We calibrate the model for by minimizing the sum of squared differences between the model and empirical autocovariance.Table D1 presents the parameters resulting from the calibration strategy and panel (a) of Figure D2 in Appendix plots the model fit to the data for when calibrated to all firms in the Portuguese economy. Two key parameters of the model are ρ_u and σ_θ, as, together, they imply that permanent heterogeneity exists. The point estimates imply that ex-ante conditions matter, as both ρ_u and σ_θ are nonzero. This again demonstrates that there seem to be differences between firms that originate from ex-ante conditions.

Finally, to more clearly identify the ex-post and ex-ante contributions, one can also derive the formula for the model autocovariance, enabling a clear identification of the contribution of both components. The autocovariance formula is given by

$$Cov[\ln n_{i,a}, \ln n_{i,a-j}] = \underbrace{\left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right)\sigma_\theta^2 + \rho_u^{2(a+1)-j}\sigma_{\hat{u}}^2 + \rho_v^{2(a+1)-j}\sigma_{\hat{v}}^2}_{\text{Ex-ante component}}$$

$$+ \underbrace{\sigma_\epsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \sigma_z^2 \mathbf{1}_{j=0}}_{\text{Ex-post component}}$$

Table D1: Calibrated model parameters for the statistical model

	ρ_u	ρ_v	ρ_w	σ_θ	σ_u	σ_v	σ_ϵ	σ_z
Portugal	0.425	0.799	0.904	0.369	0.748	0.708	0.305	0.185
United States	0.274	0.854	0.946	0.533	1.484	0.693	0.286	0.286

Note. Equally-weighted minimum distance estimates of the model in Equation (9). We use an unbalanced panel for the parameter estimates in Portugal. The parameter estimates in the United States are obtained from Pugsley et al. (2021), Appendix B, Table B.2 and are given as a reference.

and its derivation can be found in Pugsley et al. (2021). The autocovariance is a function of variance and persistence parameters of both ex-ante and ex-post shocks, as described above. Panel (b) of Figure D2 illustrates the importance of the ex-ante component for the variance as a function of a firm's age. The ex-ante component contribution is above 80% at birth and remains high even after year ten. In conclusion, all the empirical evidence in this section suggests that ex-ante heterogeneity matters both in the short and in the long-run. This may be indicative that unconstrained firms start closer to their steady state level of employment, while firms that still need to grow have a large elasticity to credit supply and so are constrained. This mechanism is mirrored in our general equilibrium firm dynamics model in Section 4.

References

Khan, A. & Thomas, J. K. (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6), 1055–1107.

Pugsley, B., Sedláček, P., & Sterk, V. (2021). The nature of firm growth. *American Economic Review*, 111(2), 547–79.