Aggregate implications of the joint size-financial constrained firm distribution*

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Abstract

Using a unique dataset covering the universe of Portuguese firms and their credit situation between 2006 and 2017 we show that financially constrained firms are found across the entire firm size distribution, even in the top 1%. Incorporating a richer productivity process that is empirically supported into an otherwise standard heterogeneous firms model generates a joint distribution of size and binding credit constraints that is in line with the data. The presence of large constrained firms in the economy, together with the fact that these firms present a higher capital elasticity to financial shocks, explains 66% of the aggregate response of output to a financial shock. We conclude the paper by providing micro-evidence in support of the model mechanisms.

Keywords: Firm size, business cycle, financial accelerator

JEL Codes: E62, E22, E23

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1 Introduction

A substantial amount of research in macroeconomics focuses on the propagation of aggregate shocks via financial factors and their relation to individual firm characteristics. In a seminal work on this topic, Gertler & Gilchrist (1994) propose firm size as an effective proxy for financial constraints. Smaller firms are arguably more risky, less liquid and face an elevated external finance premium. Accordingly, small firms are more sensitive to aggregate shocks, as they tend to be in a weaker financial position. Most heterogeneous firm models with financial frictions mirror this argument, generating a strong correlation between the size of a firm and its financial situation.

This paper provides new empirical evidence that casts doubt on this strong association between size and financial constraints. Using the Bank of Portugal's confidential credit registry database, matched with bank and firm balance sheet data between 2006 and 2017, we construct detailed, firm-specific, and credit-based measures of financial constraints. The credit registry database contains monthly information on actual, potential, short-term and long-term credit above 50 Euros extended to individuals and non-financial corporations by all financial institutions in Portugal.

Using this substantial granularity of the data we provide a novel empirical fact: financially constrained firms are found across the entire size distribution.[1] Across all our measures of financial constraints, there is a non-zero fraction of constrained firms in every size percentile. In fact, going from the bottom 5% of the size distribution to the top 5% only reduces the probability of being constrained by approximately 13% for our preferred measure.

A heterogeneous firms model with a transitory productivity process is unable to match this fact. As the productivity is mean reverting there is a relatively homogeneous optimal size for all firms. Hence, the model produces only small con-

[1] Here we measure financial constraints that are binding on the intensive margin , i.e. the maximum amount of credit that the firm can utilize, conditional on the firm's existing debt contracts. Conversely, due to data limitations, we do not consider extensive margin financial constraints, i.e. a firm's ability to get a new debt contract. For a more detailed discussion, see section 2.

strained and large unconstrained firms, contrary to our empirical findings. We resolve this counterfactual by introducing a permanent element into the firms' productivity process. This permanent component is supported by a number of empirical exercises in our data and is also corroborated by empirical findings of Pugsley et al. (2021).[2]

The rather simple addition of a permanent productivity component to the otherwise standard heterogeneous firms model enables it to generate a joint size-constrained firm distribution in line with our stylized fact, even without directly targeting any empirical moments of the joint distribution. Heterogeneity in permanent productivity introduces large and persistent heterogeneity in optimal firm sizes and spells of financial constraints. As a consequence, the model also generates a share of productive capital owned by financially constrained firms in line with the data.

We then use the augmented model to analyze how the existence of large constrained firms shapes aggregate outcomes. Whilst the aggregate responses are relatively unaffected following a productivity shock, financial shocks are greatly amplified by the presence of large constrained firms. We do this in three steps. First, we show that the top 10% of constrained firms, in terms of size, account for close to two thirds of the overall aggregate response of output and investment. As the elasticity of constrained firms to the shock is constant across the size distribution, the fact that the large firms account for a higher share of productive capital in the economy explains why the response of aggregate output and investment is so dependent on the response of large constrained firms.

Second, we proceed by showing that an heterogeneous firms model that cannot generate large constrained firms, such as a model with only a transitory productivity component, would underestimate the impacts of a financial shock on

[2] We establish the presence of a permanent productivity component using a variety of approaches. Firstly, the standard deviation of employment across firms is high and increasing with age, implying large size differences early in the life cycle and a wide range of optimal firm sizes. Secondly, the autocorrelation of employment remains high throughout a firm's life cycle. Thirdly, we also show that these statistics are diminished for constrained firms, in line with our theoretical predictions. Finally, we confirm the importance and differential incidence of ex-ante heterogeneity using the flexible statistical model developed by Pugsley et al. (2021). Results for these can be found in C.2.

aggregate output by close to 70%.

Lastly, to have a precise quantitative estimate of the effects of financial shocks on aggregate outcomes, we directly calibrate the model to match the joint size-constrained distribution. With this calibration we show that even small shocks to the financial sector can have large aggregate impacts. In particular, to generate a drop in output and investment similar to the observed during the great financial crisis in the US, a model that matches the joint size-constrained distribution needs a shock 50% smaller than a model which is not able to generate large constrained firms.

We conclude the paper by presenting some empirical evidence in support of the model mechanism. A key factor for the amplification of the aggregate effects of a financial shock is the higher elasticity of constrained firms. The existence of large financially constrained firms and the consequential higher share of productive capital in these firms does not necessarily warrant a reassessment of the effects of aggregate shocks, if these firms do not have a higher elasticity to shocks. However, we do indeed find that the turnover and employment elasticity of financially constrained with respect to 1) the business cycle, 2) idiosyncratic Total Factor Productivity (TFP) shocks and 3) idiosyncratic financial shocks, conditional on size, is higher than the elasticity of firms in a good financial situation. Hence, financial constraints explain, at least in part, the heterogeneous elasticity across firms in support of the financial accelerator mechanism. Moreover, this channel seems to be independent of potential size channels, such as the one identified by Crouzet & Mehrotra (2020).

Overall, this paper emphasizes the importance of targeting the joint distribution of size and financial constraints in order to correctly quantify the propagation and amplification of aggregate shocks in existing financial friction models. In other words, models that ignore the existence of large constrained firms may significantly underestimate the pass-through of a tightening of financial conditions on output.

Literature. Our work follows a large literature in macroeconomics that has analyzed heterogeneous firms and financial frictions both theoretically and empirically.

Firstly, we relate to the empirical literature that assesses the differences in the cyclicality of constrained firms and the debate on how to identify these firms in the data. Gertler & Gilchrist (1994) find empirical evidence for the financial accelerator mechanism. They analyze the differential cyclical behavior of small and large manufacturing firms and interpret this as evidence for the financial accelerator. Their main assumption is that size is a good proxy for financial constraints. Sharpe (1994) detects a statistically significant relationship between a firm's leverage ratio and the cyclicality of its labor force. Employment growth at highly leveraged firms is more sensitive as they are less likely to hoard labor. This cyclicality also holds for the size dimension, implicitly confirming Gertler & Gilchrist (1994)'s evidence. Related, Gilchrist & Himmelberg (1995) find that investment still responds to cash flow even after controlling for its role for forecasting future investment opportunities, with the effect being stronger for firms without full access to the capital market.

More recently, Crouzet & Mehrotra (2020), using firm level data underlying the Quarterly Financial Reports (QFR) provided by the US Census Bureau, document that differences in size-related cyclicality only arise at the very top of the distribution, with the bottom 99.5% of firms having non-significant differences in cyclicality. Arguably, this evidence, together with the insignificance of standard financial proxies for financial constraints speaks against financial factors driving cyclicality differences.

These results are also related to Farre-Mensa & Ljungqvist (2016) findings, who suggest that typical measures of financial constraints are not associated with firms that behave as if they were constrained. Even indices that combine different firm characteristics such as the ones proposed by Kaplan & Zingales (1997), Whited & Wu (2006) and Hadlock & Pierce (2010) do not correlate well with firms that behave as financially constrained. These findings are also supported by Bodnaruk et al. (2015), who use text analysis of the 10-k financial reports to gauge if firms are constrained or not, and find a weak correlation with common constraint measures. Buehlmaier & Whited (2018) equally contribute to this literature by developing a new financial constraint measure based on text analysis.

Finally, focusing on sensitivity of monetary policy Cloyne et al. (2018) find that age and dividend payments are an empirically relevant proxy for increased sensitivity to the funds rate.

Our paper, by making use of detailed firm level credit data, contributes to this literature by reiterating that size is indeed an insufficient proxy for financial constraints. Moreover, with information on credit lines available to the firm and overdue credit, we also provide evidence that supports a broader financial accelerator mechanism that is only weakly size dependent. Our measures of financial constraints significantly increase cyclicality even when controlling for size groups.

Secondly, we contribute to the research on heterogeneous firm financial frictions models. One of the early contributions in this literature by Cooley & Quadrini (2001) shares many features with our current model. They augment an otherwise standard Hopenhayn (1992) model of heterogeneous firms with financial frictions and persistent shocks. In doing so, they are able to match the empirical facts that both smaller firms, conditional on age, and younger firms, conditional on size, are more dynamic (i.e. job creation and destruction, growth, volatility of growth and exit are all higher). Clymo & Rozsypal (2019), using administrative data, equally find that young and small firms are almost twice as cyclical than large firms. In similar fashion, Pugsley et al. (2021) highlight the importance of ex-ante heterogeneity in explaining the firm size distribution and the recent decline in firm dynamism.

Another recent instance where permanent productivity differences are important to explain the evidence is Mehrotra & Sergeyev (2020). They argue that financial frictions played a relatively minor role in unemployment increases associated with the Great Recession and that employment was reduced due to shocks that affected unconstrained and constrained firms alike. Conversely, Khan & Thomas (2013) and Jermann & Quadrini (2012) argue for the importance of financial frictions and financial shocks for aggregate dynamics, respectively. Our theoretical contribution emphasizes the importance of permanent productivity differences for matching the observed distribution of financially constraint firms, conditional on size. We also highlight the importance of matching this distribution in amplifying both productivity and financial shocks, based on a model similar to the literature above.

Outlook. The paper is structured as follows. Section 2 presents the data we use for the empirical analysis and to discipline our theoretical model, as well as the novel stylized fact. In section 3 we set out the model to incorporate and account for this novel fact and in section 4 we discuss model predictions of aggregate effects. Section 5 presents micro evidence in support of the model mechanism. Finally, section 6 concludes.

2 Data

We draw on a unique combination of datasets that cover the Portuguese economy between 2006 and 2017, all managed by the Bank of Portugal Microdata Research Laboratory.

The *Informação Empresarial Simplificada* (IES) Central Balance Sheet Database is based on annual accounting data of individual firms. Portuguese firms have to fill out mandatory financial statements in order to comply with their statutory obligation. Consequently, this dataset covers the population of virtually all non-financial corporations in Portugal from 2006 onwards. We combine this dataset with the Central Credit Register (CCR) which contains monthly information on the actual and potential credit above 50 Euros extended to individuals and non-financial corporations, reported by all financial institutions in Portugal.[3] Actual credit includes loans that are truly taken up, such as mortgages, consumer loans, overdrafts and others. Potential credit encompasses all irrevocable commitments to the subject that have not materialized into actual credit, such as available credit on credit cards, credit lines, pledges granted by participants and other credit facilities.[4] We then merge these two databases on the firm level. Moreover, we also add the Monetary Financial Institutions Balance Sheet

[3]Given that the firm balance sheet data is annual, we consider the month in which the balance sheet data was reported. Results were robust to shifting and averaging the monthly credit data.

[4]Further details on the credit information used are also documented in appendix A.

Database in order to gain information on the balance sheets of banks that extend credit to non-financial institutions. We merge this dataset on a firm level using the bank identifier and the share of loans extended to one firm to arrive at our final dataset.

Similar to Buera & Karmakar (2019), who use the same dataset, we restrict the set of firms in this panel dataset to those with at least five consecutive observations and to firms which are in business at the time of reporting. Furthermore, we only consider privately or publicly held firms and drop micro firms, i.e. those with overall credit amounts of less than €10,000. Descriptive statistics for the relevant variables can be found in Table 4 in Appendix B.

2.1 Measures of financial constraints

Based on the credit information data we construct several binary measures indicating whether a firm is financially constrained or not. Financial constraints are most commonly conceived as a supply side phenomenon. Firms that could potentially obtain credit in perfect credit markets are unable to do so due to asymmetric information considerations on the supply side. For example, a firm that has a profitable investment project that requires external financing cannot realize it as the bank is not satisfied with the creditworthiness of that firm. This may happen either via the price dimension, i.e. a risk premium on the interest rate, or on the quantity dimension i.e. the credit is denied altogether. In this paper, due to data availability, we focus on the quantity dimension.[5]

Furthermore, we consider quantity-based financial constraints along two major dimensions. 1) Extensive margin financial constraints, i.e. the ability of a firm to to get a new debt contract; 2) Intensive margin financial constraints, i.e. conditional on the existing debt contracts, how much credit can a firm still use. To capture extensive margin financially constrained firms we would need to observe firms that were denied for credit. Due to data limitations, we cannot exactly identify constrained firms along the extensive margin dimension. As such, we take advantage of the very detailed credit information to construct several bi-

[5] See for example Custodio et al. (2021), or Cavalcanti et al. (2021) on how the price dimension affects firms' investment and employment.

nary measures to capture firms' credit situation on the intensive margin. These are firms that for example had credit lines open and exhausted all the credit they had available.

Measures. Many existing models classify firms as constrained if they have exhausted their maximum borrowing capacity. In our data, the closest counterpart to this metric is potential credit, summarizing irrevocable commitments by credit institutions. However, even though this measure enables an understanding of whether firms have drawn down their credit lines and may have short term liquidity needs that they cannot satisfy, it also encompasses a lot of noise. One problem might be that although firms have exhausted their committed credit lines, they could still successfully apply for a short- or long-term loan. Again, to focus on the intensive margin of financial constraints, in our baseline definition we consider a firm to be financially constrained at time t, if it has no potential credit available and neither its short- nor long-term credit (i.e. effective credit) is growing:

$$\text{Constrained I} := \mathbf{1}_{\text{Potential credit}_t=0\ \&\ \Delta\text{Effective credit}_t\leq 0}.$$

The second measure classifies firms as constrained if they don't have any potential credit available and overdue credit is positive:

$$\text{Constrained II} := \mathbf{1}_{\text{Potential credit}_t=0\ \&\ \text{Overdue credit}_t>0}.$$

The rationale behind this definition is that having overdue credit is likely a signal for a firm that exhausted all the available credit on the intensive margin and the only form of "credit" it has remaining is not paying back the existing debt. The third measure is even stricter and considers firms as constrained only if overdue credit is increasing:

$$\text{Constrained III} := \mathbf{1}_{\text{Potential credit}_t=0\ \&\ \Delta\text{Overdue credit}_t>0}.$$

While the measures presented so far are conceptually in the spirit of a firm having short term liquidity needs that it cannot satisfy and thus being in financially constrained, it might also be that a firm is in a delicate financial position if it has a large share of their credit to repay within a short period of time. The fourth measure considers this possibility by classifying a firm as constrained if the share of credit to assets that is due within the next year is in the top 10 percent of the distribution:

$$\text{Constrained IV} := \mathbf{1}_{\frac{\text{Credit} < 1 \text{ Year Maturity}_t}{\text{Total Assets}_t} > \mathbf{P}_{90}}.$$

Our final measure follows the evidence presented by Rampini & Viswanathan (2020) that financially constrained firms use more secured debt, and considers a firm to be financially constrained if the share of secured debt over total assets is in the top 10 percent of the distribution:

$$\text{Constrained V} := \mathbf{1}_{\frac{\text{Secured Debt}_t}{\text{Total Assets}_t} > \mathbf{P}_{90}}.$$

Appendix A provides a more detailed description of the dataset and the underlying variables for the constraints measures. Table 6 reports the correlation matrix between the different measures. Finally, Figures 7 and 8 report the evolution of the share of constrained firms and credit over time.

2.2 Financial constraints along the empirical firm distribution

Utilizing our measures of financial constraints, we present a new stylized fact: Size, and other measures typically used as proxy for financial constraints, are only weakly correlated to the firm's financial health. In fact, financially constrained firms can be found over the entire firm size distribution. Figure 1 plots the share of firms that have zero potential credit and no increase in effective credit (measure I) over percentiles of total assets, age, liquidity ratio and leverage. Evidently, financially constrained firms can be found in every bin of the firm distribution. In particular, there are constrained firms across the entire firm size distribution, as illustrated by the plot over percentiles of total assets. This finding is robust across all binary identifiers for being constrained, with only the overall fraction of firms with poor financial health changing depending on the strictness of the specific measure, as documented in Figures 9 - 12 in Appendix C.1.

While correlations are in line with the existing literature, they are not as strong as existing theory might predict. In fact, when running a linear probability model, the probability of being constrained only reduces by about 13% for two standard deviation increase in total assets, which is equivalent to going from the bottom 5% to the top 5% of the size distribution.[6] Even after accounting for potential attenuation bias, the main conclusion stands: a firm dynamics model with just transitory productivity shocks typically produce exclusively small constrained firms and large unconstrained firms, yet our data does not support this strong dichotomy. Moreover, even when controlling for a battery of financial variables the explanatory power to predict whether a firm is constrained or not is relatively low compared to the firms' fixed effects. Hence, existing proxies of financial constraints may be unable to capture this unobserved heterogeneity, which seems to play a substantial role in credit decisions.

3 Model

In this section we present a heterogeneous firms model with financial frictions which aims to replicate the stylized fact of section 2.2. We build on Khan & Thomas (2013) and introduce ex-ante heterogeneity through a permanent productivity component which can be interpreted as the firm's business potential. We find empirical evidence in support of the inclusion of a permanent productivity component using a variety of approaches, following Pugsley et al. (2021). Firstly, the standard deviation of employment across firms is high and increasing with age, implying large size differences early in the life cycle and a wide range of optimal firm sizes. Secondly, the autocorrelation of employment remains high throughout a firm's life cycle. These two results point towards the importance of permanent firm differences. Thirdly, we also show that these statistics are diminished for constrained firms, in line with our theoretical predictions. Finally, we confirm the importance and differential incidence of ex-ante heterogeneity using the flexible statistical model developed by Pugsley et al. (2021). For more

[6] See Table 5 in Appendix B for the results of the linear probability model.

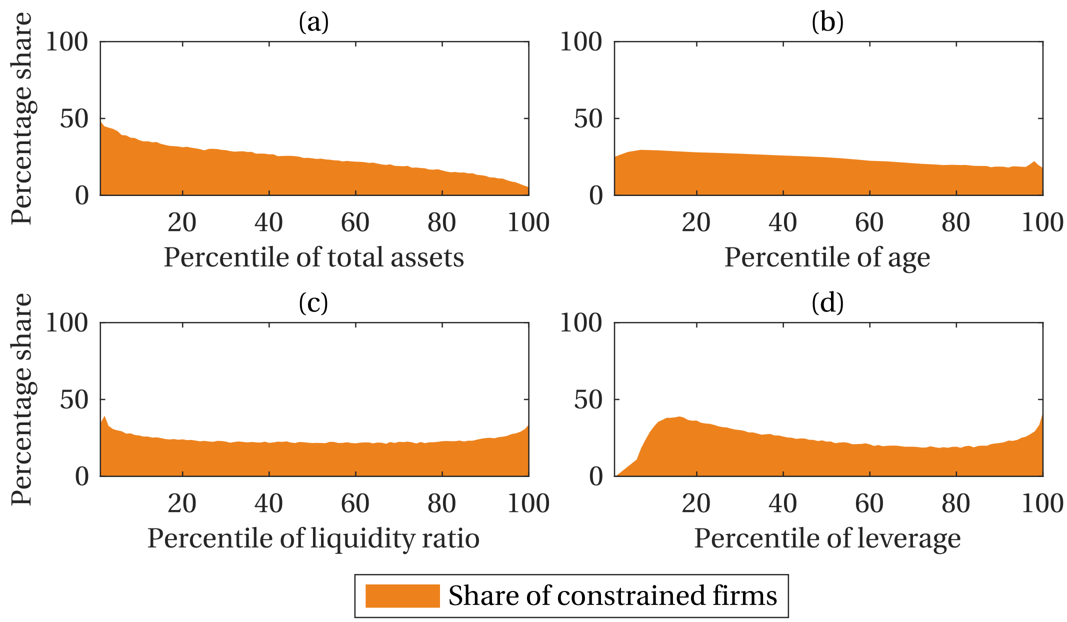

Figure 1: Decomposition of constrained and financially healthy firms across percentiles of firm variables. Financially constrained firms are identified using measure I which classifies firms as constrained if they have exhausted their potential credit and were not obtaining additional credit in that period.

details, check Appendix D.

The simple addition of a permanent productivity component breaks up the strong correlation between size and financial constraints. Firms with lower permanent productivity will reach their optimal amount of capital and will be unconstrained from then on, while firms which draw a higher permanent component may be constrained even when very large as they are still growing to reach their high potential.

3.1 Households

The household sector of the model is deliberately simple. In particular, a representative household chooses consumption, savings and labor supply according to the following maximization problem:

$$V(k) = \max_{c,l,k'} \left\{ U(c,l) + \beta \mathbb{E} V(k') \right\}$$

subject to:

$$k' + c = (1+r)k + \omega l + D,$$

where c is consumption, l is labor, k is capital and D are dividends. ω is the wage, r is the real interest rate. The first-order conditions for the household problem are standard:

$$U_l(c,l) = \omega U_c(c,l)$$
$$U_c(c,l) = \beta \mathbb{E}\left[(1+r')U_c(c',l')\right].$$

We use the following Greenwood-Hercowitz-Huffman (GHH) utility formulation:

$$U(C,N) = \log(C) + \psi(1-N)$$

Consequently, in the absence of aggregate risk, the first-order conditions are:

$$(1+r) = \frac{1}{\beta}$$
$$\omega = \psi C$$

3.2 Production

The production sector features a continuum of firms, indexed by i. Firms are either classified as entrants or incumbents, as detailed below.

Incumbents. An incumbent firm i produces according to the following production function:

$$y_i = \varphi_i k_i^{\alpha} l_i^{v}, \quad \alpha + v < 1,$$

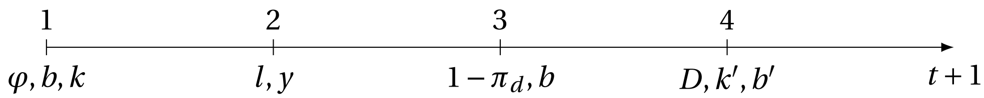

Figure 2: Within period timing of incumbent firm

where k and l are capital and labor inputs and φ denotes idiosyncratic productivity. Every firm's productivity comprises two components:

$$\ln \varphi_i = w_i + \theta_i,$$

where w_i is an idiosyncratic *transitory* productivity shock, which follows an AR(1) process with persistence ρ_w and variance of innovations σ_ϵ^2. θ_i is the *permanent* productivity component, drawn at birth from a normal distribution with mean μ_θ and variance σ_θ^2 [7]

$$\begin{aligned}
&\theta_i \overset{\text{iid}}{\sim} \mathcal{N}\left(\mu_\theta, \sigma_\theta^2\right) \\
&w_i' = \rho_w w_i + \varepsilon_i \quad \varepsilon_i \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \quad \left|\rho_w\right| \leq 1
\end{aligned}$$

The firm's total profits before investment are revenue minus labor costs (in what follows we suppress i, the firm indicator, to ease on notation where possible):

$$\pi = y - \omega l,$$

where ω is the wage per unit of labor.

Figure 2 summarizes the within-period timing of the incumbent. The firm enters the period with predetermined levels of debt b and capital k and immediately observes its idiosyncratic productivity φ composed of a permanent and transitory component. Next, the firm's labor decision is a static choice that can be found through the firm's first order condition:

$$l(k, \varphi; \omega) = \left(\frac{\upsilon \varphi}{\omega} k^\alpha\right)^{\frac{1}{1-\upsilon}}.$$

[7] Henceforth, when we refer to a model with only a transitory shock we mean that $\ln \varphi_i = w_i$.

After the production stage, the firm may suffer an exogenous exit shock. The shock happens with probability π_d. Consequently, the value of the firm after the production stage is given by

$$V^1(x,\varphi) = \pi_d x + (1-\pi_d)V^2(x,\varphi)$$

If the firm survives the exit shock, at the end of the period it chooses debt b' and capital k' to take to the next period and dividends to distribute this period D to maximize its value

$$\begin{aligned} V^2(x,\varphi) = \max_{k',b',D} & \left[D + \mathbb{E}_{\varphi'|\varphi} \Lambda V^1(x',\varphi')\right] \qquad (1) \\ \text{s.t.:} & \\ & D \equiv x + qb' - k' \geq 0 \\ & b' \leq \xi x \\ & x' \equiv x(k',b',\varphi') = y(l(k',z'),k',\varphi') - wl(k',\varphi') + (1-\delta)k' - b' \end{aligned}$$

where ξ is the financial parameter that captures the financial frictions in the economy, x is the net worth with which the firm starts the period, given as the sum of profits plus the value of the non-depreciated capital minus the debt the firm has to pay back. q is the price of the bonds firms issue, with $\frac{1}{q}-1$ equal to the equilibrium interest rate, r. Λ is the firm discount factor. As the representative household is the owner of the firm, we assume $\Lambda = \beta$ in steady state.

The firm faces two critical constraints according to (1). First, the firm cannot issue negative dividends or, equivalently, raise equity. Second, the firm is only able to borrow up to an exogenous fraction ξ of its total cash on hand. We opt for a cash on hand collateral constraint following evidence from Kermani & Ma (2020) or Lian & Ma (2021), which illustrates firms' debt contracts and financial constraints do not depend solely on assets, but also on the firm's value and cash flow. Our measure of cash on hand captures exactly these two sides, as it takes into account both the cash flow and the non-depreciated capital.

Entrants. Entry in this model is exogenous. We assume there is a fixed measure, M_e, of entrants equal to the mass of firms exiting after receiving a death shock. The entrants are assumed to enter with zero debt ($b_0 = 0$) and are log normally distributed over their initial capital k_0 with the mean anchored at a fraction of the mean of optimal capital levels. The choice of a log normal distribution is motivated by the right skewed distribution of entrants in the data. The initial productivity of each entrant, φ_0, follows the same process as the incumbents' productivity. Note that firm entry takes place at the end of a period, and entrants start operating in the next period, given their initial state, (k_0, b_0, φ_0).

3.3 Firm level decisions

To characterize the firms' decisions we divide the firms into three groups, following Khan and Thomas (2013). This simplifies the solution of the model significantly.

1. **Unconstrained firms.** Firms that can implement the optimal amount of capital and guarantee that in the future they will never be constrained again.

2. **Constrained firms, type 1.** Firms that can implement the optimal amount of capital but not the minimum savings policy that guarantees they will never be constrained again in the future.

3. **Constrained firms, type 2.** Firms that are constrained and cannot implement the optimal amount of capital nor the minimum savings policy.

For model details on the decisions of the firms in each group see Appendix F.

3.4 Simplified model predictions

The way in which firms respond to different types of shocks will ultimately depend on whether they have reached their optimal amount of capital or whether they are still growing. Hence, in what follows, we refer to firms which can implement their optimal capital level as being unconstrained and otherwise as constrained. Consequently, type 1 constrained firms are considered unconstrained as they can implement the optimal amount of capital and their investment policy is the same as for unconstrained firms if shocks are relatively small.[8]

To gain more insight into the respective investment elasticities to aggregate shocks and the role of ex-ante heterogeneity, we consider a slightly simplified version of the model as outlined in Appendix G. In this model we abstract from labor and assume there is no uncertainty except for a stochastic death shock. The main intuition about differential investment elasticities is captured in Proposition 1.

Proposition 1 *Constrained firms are more elastic to an aggregate TFP shock than unconstrained firms, absent any cyclicality in the constraint, if*

$$mpk_t > \rho \frac{\alpha}{1-\alpha} \frac{1}{1+q_t \xi}.$$

Proof: The proof is provided in Appendix G.

Constrained firms will only respond more to an aggregate productivity shock if either their marginal product of capital is large enough, i.e. they are far from their potential, or if the aggregate shock is quickly fading (ρ is close to 0) which gives unconstrained firms little incentive to adjust their capital amount, as productivity is quickly mean reverting. In fact, the elasticity of unconstrained firms is independent of their potential. On the other hand, the marginal product of capital of constrained firms is higher the higher their potential and the farther they are from reaching their potential.

Hence, the overall aggregate response of output and capital depends on the distribution of constrained firms across the firms size distribution. Furthermore, the financial accelerator mechanism will only be present in the model economy, if Proposition 1 holds on average. In our discussion about aggregate implications below, we separately consider the case of a temporary aggregate shock to total factor productivity (TFP) and a credit shock as a negative shock to borrowing conditions that revert to the steady state value after 1 period.

[8] Large shocks could make the constraint bind again, and they would become strictly constrained.

3.5 Solving and calibrating the model

Solution Method. As outlined in Subsection 3.3, one can categorize firms into constrained, potentially constrained and unconstrained firms. The two cash-on-hand thresholds that define to which group a firm belongs are derived in Appendix F. One can then directly solve for the capital and bond policy function numerically.

To solve for the general equilibrium, we approximate the firm distribution over a fixed grid of net worth using the histogram method proposed by Young (2010).

The steady state solution is then given at the wage which is leading to a clearance of the goods market.[9] Given the steady state wage, we also conduct a Monte Carlo simulation to study the firms' policy responses to aggregate shocks in partial equilibrium.

Calibration. For most of the parameters, which are unrelated to distributions in the model, we follow Khan & Thomas (2013). The set of parameters chosen is documented in the upper part of Table 23 in Appendix B. The discount factor, β, is set to yield an average annual real interest rate of 4%. The production parameters, η and α, imply a labor share of 60% and capital share of 30%, respectively. Leisure preferences imply that households work one third of their available time.

Firm exit rates in the data are heterogeneous and tend to be lower for larger and older firms. In order to account for this without introducing a size based exit rate schedule, we compute a size weighted average exit rate. When not accounting for lower exit rates among performing firms, small firms with high potential are likely to drop out prior to reaching their optimal amount of capital.[10]

The mean productivity levels for the permanent and transitory component, μ_θ and μ_w, are normalized such that when transforming them into a log-normal distribution, the average productivity component equals one.[11] The rest of the

[9] Market clearing interest rates are given by $1/\beta$ due to the household's first-order condition.

[10] The model can still fit the data reasonably well for higher exit rates, yet it gets harder to match the skewness of the firm size distribution as firms with high potential and a long growth path are proportionally more likely to exit before they reach their full size.

[11] Note that the mean of a log-normal distribution is affected not only by the location param-

Table 1: Calibrated model fit

Moment	Data	Model
Size of 90th percentile / median	9.440	9.218
Average leverage	0.626	0.330
Std of value added	1.559	1.644
1-year autocorr value-added	0.924	0.928
5-year autocorr value-added	0.818	0.762
Std of value-added growth	0.382	0.384
% constrained firms	0.244	0.250

Notes. All constrained firms moments are calculated using constrained measure I.

parameters - collateral constraint ξ, standard deviation of permanent shock σ_θ, persistence and standard deviation of the transitory shock ρ_w and σ_w, and the relative size and standard deviation of entrants μ_{ke} and σ_{ke} - are calibrated using the simulated method of moments (SMM).

The values presented in the lower part of Table 23 in Appendix B minimize the distance between a set of empirical moments of the firm distribution. The moments chosen are commonly targeted in the literature to discipline the distribution of firms along the size dimension, and the life cycle of the firm, namely in terms of the speed at which firms grow and reach their optimal size.[12] Additionally, we target the % of constrained firms in the data according to measure I.

Table 1 compares the targeted moments in the data and in the model.

Non-targeted moments. On top of the model fitting well the targeted moments, it equally generates a joint size-constrained firm distribution in line with the data. As documented in the far right column of Table 2, the model generates both large constrained firms and small unconstrained ones. As a consequence, it is also generating a share of assets in constrained firm close to the empirically

eters but also the scale parameter. We adjust it accordingly, such that for any scale parameter, $\mu = 0$ yields an average productivity of 1, when transformed to a log-normal.

[12] See for example Midrigan & Xu (2014) or Khan & Thomas (2013). We use value-added for some of the moments as it is the closest counterpart to revenues in the model given that we abstract from intermediate goods.

Table 2: Untargeted moments

Moment	Data	Model
Share of const. firms in bottom 20%	0.33	0.65
Size of const. firms 90th percentile / median	7.35	9.72
Size of unconst. firms 90th percentile / median	9.67	9.05
Asset share of const. firms	0.07	0.10
Share of const. firms in top 10% vs. bottom 20%	0.36	0.05
Percentage of const. firms in top 1%	0.09	0.01

Notes. All constrained firms moments are calculated using constrained measure I.

observed values.

In section 2.2 we highlight that constrained firms are found across the entire distribution of firms. As illustrated in Figure 1, even at the top of the distribution in terms of size close to 10% of the firms are constrained.

Figure 3 compares the model generated share of constrained firms across the size distribution with its empirical equivalent. The model generates small unconstrained firms as well as large constrained firms and is also able to approximate the untargeted deciles of the empirical distribution quite well, as depicted on the figure. It still slightly overestimates the share of constrained firms at the bottom of the distribution, and underestimates the share of constrained at the top, but the remaining deciles are close to the data counter-part.

The distribution generated by the model is explained by the fact that some larger firms are still growing to reach their steady state capital and are still constrained. At the same time, the model accounts for a larger share of small firms that are born at or close to their steady state level of capital.[13]

4 Discussion

In this section we assess the implications of accounting for large constrained firms when faced with an aggregate financial shock, respectively.

[13] Figure 18 in Appendix C.3 offers a slightly different perspective, plotting the density distribution of constrained and unconstrained firms. It is possible to observe that the distributions of constrained and unconstrained overlap.

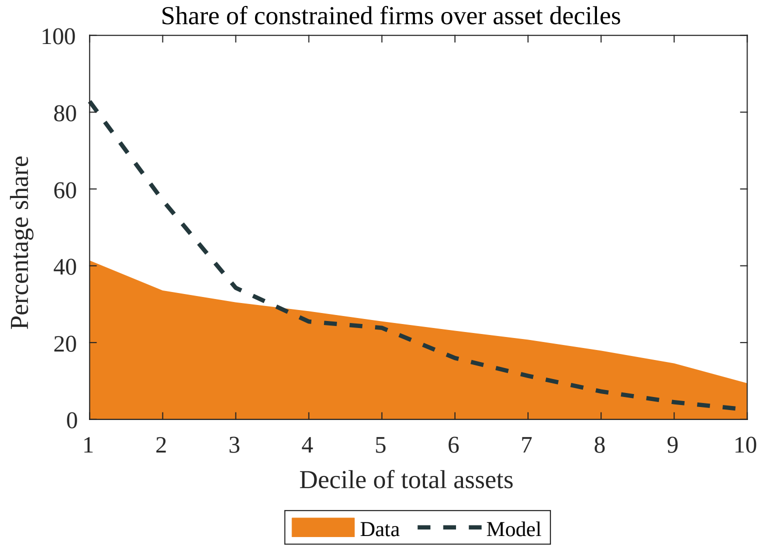

Figure 3: Share of constrained firms across the distribution. Empirically, constrained firms are identified using measure I which classifies firms as constrained if they have exhausted their potential credit and were not granted additional short- or long-term credit in that period.

We now proceed to assess the aggregate implications of accounting for constrained firms across the entire firm size distribution. We assume a drop in the maximum borrowing capacity of 50%.[14] Given the sudden and transitory nature of the financial shock, we assume wages to be fixed at the general equilibrium level before the shock hits.[15]

Figure 4 shows the responses to the credit shock depicted in the upper left panel. Since the firm's capital stock is pre-determined, there is no direct impact in period $t = 2$, when the financial shock hits. However, the lower maximum borrowing capacity affects constrained firms in their investment decision, while unconstrained firms remain unaffected by the shock as their borrowing constraint is not binding.

[14] Khan & Thomas (2013) simulate an 88 percentage point drop in ξ. However, in their calibration the initial level of ξ is 1.38. In our calibration ξ is 0.50, hence a 50% drop equals a 25 percentage point drop in maximum borrowing allowances.

[15] General equilibrium results for this exercise lead to the same qualitative conclusions, but we prefer the partial equilibrium analysis to isolate the effect coming from the differences in the distribution of constrained firms.

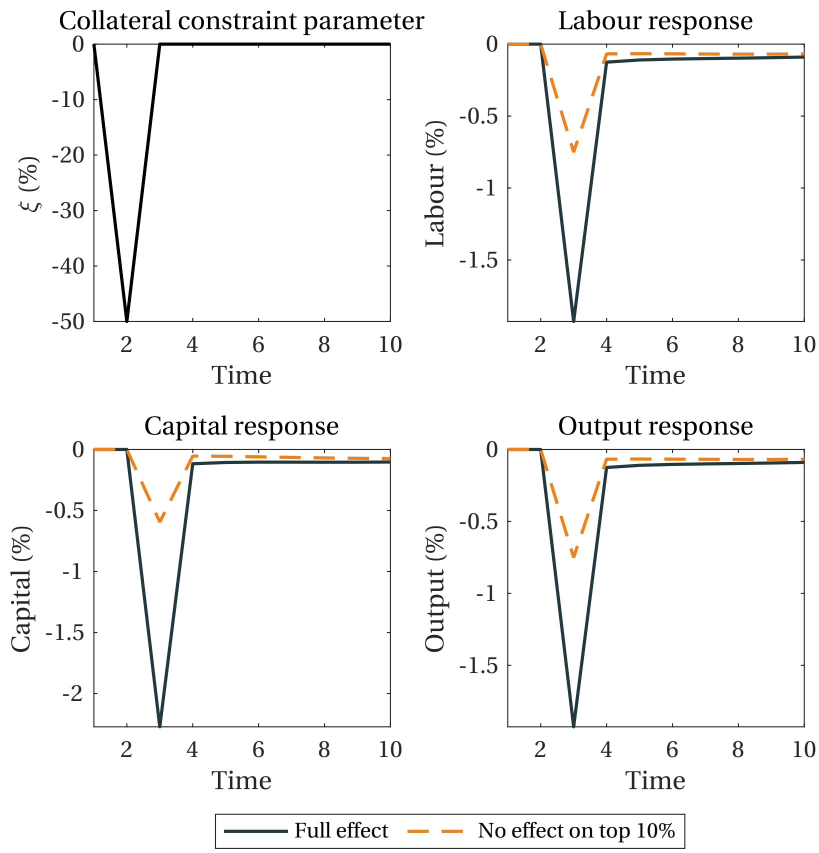

Figure 4: IRFs to a financial shock. Lines indicate the partial equilibrium response to a shock to ξ in the upper left panel, with wages fixed at their steady state level.

The resulting aggregate effect of constrained firms reducing their investment depends heavily on the distribution of these constrained firms along the firm size distribution. The dark blue line in Figure 4 illustrates the overall effect of a financial shock on labour, capital and output. Output drops by close to 2% in response to a 50% decrease in the collateral constraint parameter. The dashed orange line shows the aggregate responses in the scenario that this shock does not affect the top 10% of constrained firms. The overall effects are reduced by approximately 60%, with output only dropping 0.75% in this scenario. This illustrates that the top 10% of constrained firms account for more than half of the overall decrease in output in response to a financial shock.

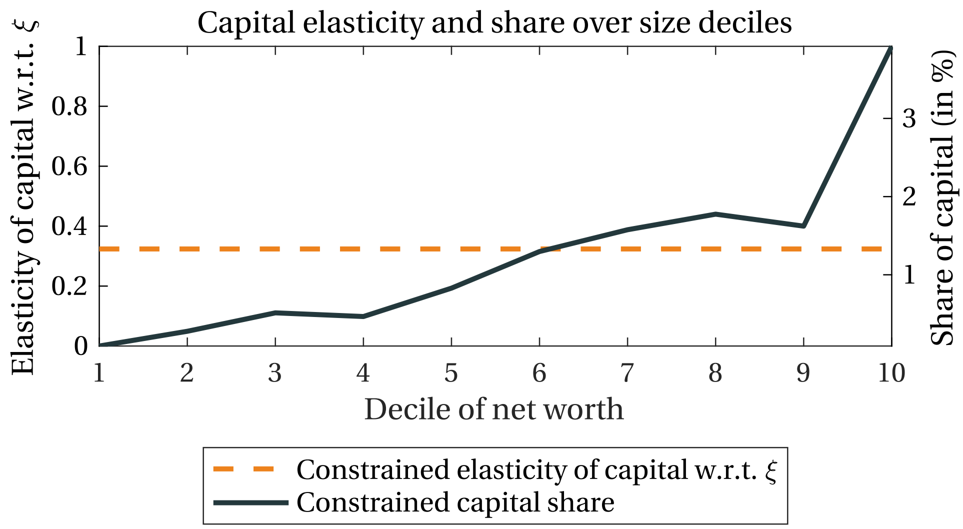

Figure 5: Elasticity of constrained firms with respect to ξ and share of capital in constrained firms over the size distribution.

The quantitative magnitude of the effect clearly depends on the fraction of firms identified as being constrained by the different constrained measures ranging from 24% (Measure I) to as low as 2% (No potential credit and increasing overdue credit) of all firms. Yet, since all measures are suggestive of the notion that constrained firms exist along the entire firm size distribution, the conclusion that large constrained firms explain the majority of the fluctuations in response to financial shocks is robust to calibrating the model to any of the credit constrained measures we use in section 2.

Figure 5, illustrates the key mechanism at play. The dashed orange line presents the elasticity of constrained firms with respect to ξ, which is independent of size. For unconstrained firms, as already pointed out, the elasticity is zero. The dark blue line is indicating the share of productive capital in constrained firms by net worth decile. Despite the share of constrained firms at the top of the size distribution being smaller than at the bottom of the distribution – as shown in Figure 3 –, the percentage of productive capital in this large constrained firms is much higher. This together with the fact that the elasticity of constrained firms to the shock is independent of size, explains why large constrained firms account for around two thirds of the overall response of output to the financial shock.

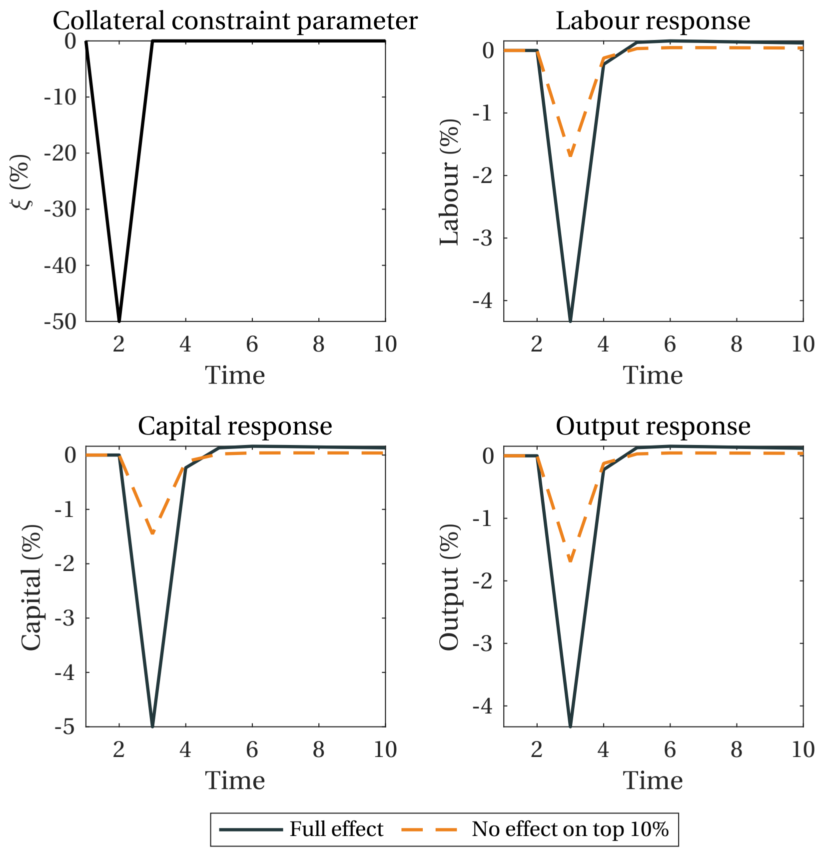

Figure 6: IRFs to a financial shock in a model that targets directly the joint size-constrained distribution. Lines indicate the partial equilibrium response to a shock to ξ in the upper left panel, with wages fixed at their steady state level.

The mechanism holds even when we consider the general equilibrium. In this case, given the drop in capital in response to the financial shock, wage decreases which causes labor and consequently output to not fall by as much as in the partial equilibrium scenario. Still, in the general equilibrium case the large constrained firms account for 70% of the overall decrease in output. The results are presented in Figure 19 in Appendix C

To have a more precise quantitative effect of the mechanism just highlighted, we re-calibrate the model to target directly the joint size-constrained firm distribution. The calibration parameters and fit can be find respectively in Tables 24 and 25 in Appendix B. With the new calibration at hand, we re-estimate the impact of a financial shock. We find that when matching the joint size-constrained distribution the effects of a financial shock can be amplified by more than twice as high of drop in output when compared to our benchmark calibration, as can be seen in Figure 6, with the large constrained firms again explaining around two thirds of the drop in output. This result highlights that even a small shock in the financial sector can lead to large aggregate effects due to the granular effects coming from large constrained firms.

To illustrate the importance of generating an empirically plausible size-constrained joint distribution, and the role of the permanent productivity component in doing so, we shut down the permanent productivity component and directly calibrate the model to target the joint size-constrained distribution. The calibration results can be found in Table 26 in the Appendix B. Figure 17 in the appendix shows that, even when directly targeting the joint distribution, a model without a permanent productivity component cannot generate large constrained firms, while the model with a permanent component closely matches the joint size-constraint distribution.

With the distribution generated by the model without a permanent productivity component, we then calculate the aggregate effects of a financial shock. In Figure 20 we can see that in the absence of large constrained firms, output drops by less than one third than in our benchmark model. This shows the importance of incorporating a permanent productivity component that allows the model to generate a joint size-constrained distribution in line with the data when assessing the quantitative effects of financial shocks in the economy.

Additionally, in Figure 19 in Appendix C.3 we consider an unexpected and temporary 1% increase in total factor productivity (TFP). In a direct response to the shock, firms employ more labor for any predetermined level of capital. While unconstrained firms do not increase their investment in capital due to the transitory nature of the shock, constrained firms leverage their increased net worth to borrow more. This explains why the lagged response in capital is much smaller than the response in labor, as only constrained firms react to the shown case of $\rho = 0$, which is only 23% of all firms in this calibration.

However, given the small magnitude of the capital response relative to the response in labor, the difference barely shows up in aggregate output. Note, the effect would become stronger if the borrowing constraint was cyclical or the fraction of constrained firms in the economy was higher.[16]

5 Mechanism validation

A higher asset share held by constrained firms and a presence of constrained firms across the entire size distribution do not necessarily warrant a reassessment of the cyclical properties of financial frictions models. At the core of the mechanism highlighted in the previous section is the fact that constrained firms have a higher elasticity to shocks. This section aims to validate this important mechanism with data. In particular, we show that, empirically, financially constrained firms are more cyclical than their counterparts, and are more responsive to both TFP and financial shocks, conditional on size. In order to illustrate this point we use a specification similar to Crouzet & Mehrotra (2020), augmented with the set of firm-specific and time-varying measures of financial factors:

$$\begin{aligned} g_{i,t} = \kappa u_{it} + \sum_{j \in \mathscr{J}} (\alpha_j + \beta_j u_t) \mathbf{1}_{i \in \mathscr{S}_t^{(j)}} + (\zeta + \eta u_t) \text{Const.} n_{i,t} \\ + \gamma_l + \delta_t + \lambda_{lt} + \alpha_i + \epsilon_{i,t}, \end{aligned} \tag{2}$$

where i identifies a firm and t identifies a year. The dependent variable $g_{i,t}$ is the year-on-year log change in turnover. The set $\mathscr{S}_t^{(j)}$ is a j^{th} size group, e.g. all firms above the 90^{th} but below the 99^{th} percentile. We include three size groups, $j \in \{[90, 99], [99, 99.5], [99.5, 100]\}$. Furthermore, $u_i t$ takes the form of three different variables: 1) the year-on-year growth rate of GDP; 2) TFP shocks at the firm level, using the method proposed by Ackerberg et al. (2015); and 3) bank level shocks aggregated at the firm level, following Amiti & Weinstein (2018). Const.$n_{i,t}$ refers to the firm-specific variable measuring financial constraints introduced in section 2, indexed by n. Finally, we also include firm α_i, industry γ_l,

[16]One should also note that the difference between the models would vanish and eventually flip if the TFP shock gets more persistent and unconstrained firms become more cyclical, as shown in proposition 1.

Table 3: Semi-elasticity of turnover conditional on size and measures of financial constraints

	Un-constrained	Constrained measure I	II	III	IV	V
% Δ GDP	2.316***	0.311***	1.495***	0.882***	0.085	-0.145
	(0.056)	(0.054)	(0.175)	(0.217)	(0.103)	(0.102)
TFP shock	0.086***	0.016***	0.076***	0.075***	0.068***	0.065***
	(0.001)	(0.004)	(0.009)	(0.010)	(0.007)	(0.008)
Fin. shock	0.054***	0.014	0.155*	0.128*	0.179***	0.073*
	(0.005)	(0.013)	(0.049)	(0.057)	(0.040)	(0.035)

Notes. Estimates report the financial constrained firms semi-elasticity of turnover relative to the control group of unconstrained firms, with respect to GDP, TFP and Financial shocks. Constrained measures are constructed as outlined in section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

year δ_t and industry year λ_{lt} fixed effects. [17]

Table 3 reports estimates of the coefficient of interest η, the semi-elasticity of firm-level growth in turnover to the different shocks relative to the control group of firms financially healthy. In the first line we have the semi-elasticity to the economic cycle, captured by GDP growth. In the second line we report the semi-elasticity relative to firm-level TFP shocks estimated as in Ackerberg et al. (2015). The third line presents the semi-elasticity to a financial shock, identified using the methodology proposed by Amiti & Weinstein (2018). The columns present the elasticity for the different constrained measures.

The first column reports the semi-elasticity of the control group, healthy firms, to the different shocks. The results can be interpreted as follows: for a 1% increase in GDP growth, firm-level TFP or credit supply, turnover of unconstrained firms increases by 2.3%, 0.9% and 0.5% respectively.

The remaining columns report the semi-elasticity of financially constrained firms, for the different measures outlined in section 2.1, relative to the healthy

[17] For the regression where u_t is GDP growth, we include an interaction of industry dummies and GDP growth instead of industry year fixed effects. We then test the robustness of the results for including industry year fixed effects.

firms. The results for our baseline measure indicate that constrained firms have, on average, a semi-elasticity to GDP growth, TFP and financial shocks, that is 0.31, 0.02 and 0.01 percentage points higher than unconstrained firms, offering support for the financial accelerator mechanism.

However, as already pointed out when introducing the different measures for being in financial constrained, the baseline measure might capture firms for which potential credit is zero, but are in fact unconstrained. Hence, the baseline measure offers a lower bound of the increased sensitivity of firms in poor financial shape. We therefore consider the other binary measures trying to overcome these drawbacks, reported in columns II to V. These estimation results are supportive of the notion that the baseline measure acts as a lower bound and that the sensitivity might be up to one order of magnitude higher for constrained firms, as measured by measure II.

This evidence is in line with growing literature on the causal effect of financing constraints on firm level outcomes. Financially constrained firms are found to have a higher elasticity of investment and employment with respect to shocks to the collateral value (Gan, 2007; Chaney et al., 2012), to financial shocks (Chodorow-Reich, 2014) and to monetary policy shocks (Greenwald et al., 2019; Ottonello & Winberry, 2018). The results presented in this section are in line with the literature results and suggest that our financial measures are indeed capturing the firms in poor financial shape.

Results for the remaining regression coefficients are presented in Tables 7, 8 and 9 in Appendix B. It is worth noting that the estimation coefficients with respect to size groups hardly change when including the different financial measures. This is indicative of the fact that the mechanism going through size is somewhat independent to any financial accelerator mechanism and that size might not be a good proxy for the latter, as already pointed out by Crouzet & Mehrotra (2020).

In Tables 10 to 12 in Appendix B we present the results when considering growth in employees instead of turnover. Besides using different measures, we also consider a battery of robustness checks for our GDP, TFP and financial shocks regressions. First, we exclude firm fixed effects. Second, in the GDP regression, we include time fixed effects to account for broader macroeconomic circumstances. Third, we estimate the model excluding those firms for which potential credit is zero throughout. Fourth, we control for supply effects using aggregated bank data. Estimates are robust across all specifications and the results can be found in Tables 13-22 in Appendix B.

6 Conclusion

This paper documents a novel empirical fact: at any point of the firm size distribution there are financially constrained firms. This is counterfactual to the predictions of an heterogeneous firm financial frictions model with only transitory productivity shocks. We subsequently analyze the importance of matching this fact in a quantitative financial frictions model with heterogeneous firms.

Next, we build a standard firm dynamics model, with a richer productivity, for which we find empirical evidence in support. We demonstrate that by adding a permanent component to the productivity process helps the model generate a joint size-constrained firm distribution in line with the data, breaking the typical strong correlation between financial constraints and size and generating a large mass of small unconstrained and large constrained firms. The existence of large constrained firms consequently drives up the share of productive capital in this type of firms. This, together with the fact that constrained firms are more elastic to financial shocks, has significant implications for aggregate responses to financial shocks. The effects of a financial shock are strongly affected by the presence of large constraint firms, with the largest 10% of constrained firms explaining two thirds of the output drop in response to a financial shock.

We conclude the paper by presenting empirical evidence in support of the model mechanism. We show empirically that constrained firms present higher elasticity to financial shocks, which is a key driver of the larger aggregate response of capital, employment and output to this type of shocks.

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A Variable definitions

Central Credit Responsibility Database (Central de Responsabilidades de Crédito)

Identifier (tina): Anonymized tax identification number.

Global Credit (valor_global): is the sum of effective credit and potential credit, representing the total available credit that a firm accesses.

Effective Credit (valor_efectivo): is credit effectively used in a regular situation, i.e., without payment delays as defined in the respective contract. Examples of effective responsibilities are:

- Loans for the acquisition of financial instruments (shares, bonds, etc.);
- Discount and other credits secured by effects;
- Overdrafts on bank accounts;
- Leasing and factoring;
- Used amounts of credit cards.

Potential Credit (valor_potencial): represents irrevocable commitments of the participating entities. Banco de Portugal requires all credit-granting institutions to report to the CCR their outstanding loan exposure by instrument of all irrevocable credit obligations. Examples of potential responsibilities are:

- Unused amounts of credit cards;
- Lines of credit;
- Guarantees provided by participating entities;
- Guarantees and guarantees given in favor of the participating entities;
- Any other credit facilities likely to be converted into effective debts.

Overdue Credit (valor_vencido): All outstanding credit exposures recorded as non-performing (including overdue, written off, renegotiated credit, overdue credit in litigation, and written off credit in litigation) are aggregated to calculate overdue credits. It includes principal, interest and related fees.

Short-term Credit (valor_curto): Short-term credit is calculated using two different definitions. In the first place, short-term credit is defined based on the term-to-maturity as agreed in the credit contract, denoted by valor_curto_o. Specifically, short-term credit has original maturity of equal to or less than one year. Before 2009, the CCR dataset did not streamline credit exposure based on the maturity structure. Therefore, for the data before 2009, the short-term credit is defined as the aggregation of commercial credit, discount funding, and other short-term funding, which are short-term funding by their nature. In the second place, short-term credit is defined based on residual maturity – the remaining time until the expiration or the repayment of the instrument, denoted by valor_curto_r. Specifically, it is credit with residual maturity of equal to or less than one year. This variable is only available from 2009 onwards. Potential credit is excluded for both calculations.

Long-term Credit (valor_longo): Similar to short-term credit, long-term credit is defined based on original and residual maturities. More precisely, long-term credit is credit with an original or residual maturity of more than one year, denoted by valor_longo_o and valor_longo_r, respectively. Long-term credit defined on an original maturity basis (valor_longo_o) for the data before 2009 is the aggregation of total credit excluding commercial credit (type 1), discount funding (type 2), and other short-term funding (type 3). Potential credit is excluded for both calculations.

B Additional tables

Table 4: Descriptive statistics of Portuguese firms between 2006 and 2017

Variable	Mean	Median	Std. Dev.	Size group median <90th	90th-99th	99-99.5th	>99.5th
Total Assets	3.15	0.28	85.10	0.25	5.06	42.71	135.70
Turnover	1.86	0.23	33.59	0.21	3.25	19.93	27.94
Potential credit	0.19	0.03	4.56	0.03	0.14	0.95	2.95
Effective credit	0.53	0.04	5.96	0.04	1.15	6.93	126.73
Leverage	0.28	0.20	0.38	0.20	0.24	0.17	0.08
Liquidity ratio	0.14	0.06	0.19	0.06	0.02	0.01	0.01
Age	15.01	12.00	12.26	12.00	21.00	23.00	21.50
Employees	14.47	4.00	130.58	4.00	25.00	95.00	98.00
# Banks	2.45	2.00	1.89	2.00	4.00	4.00	5.00

Notes. Total assets, turnover, potential credit and effective credit are measured in 2010 Euro Millions.

Table 5: Linear probability regression: How age, total assets, leverage and liquidity ratio affect the probability of being constrained according to measure I

	Constrained binary (1)	(2)	(3)	(4)
Age	-0.034***			
	(0.000)			
Total assets		-0.066***		
		(0.000)		
Leverage			-0.008***	
			(0.000)	
Liquidity ratio				0.007***
				(0.000)
Constant	0.246***	0.245***	0.244***	0.244***
Observations	1,365,913	1,365,913	1,365,913	1,365,913
R-squared	0.006	0.024	0.015	0.000

Notes. Here we use winsorized response variables at the 99.5th and 0.5th percentile.
Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 6: Correlation between different measures of financial constraints

Measure	I	II	III	IV	V
Constrained I	1				
Constrained II	0.306***	1			
Constrained III	0.259***	0.812***	1		
Constrained IV	-0.034***	0.062***	0.052***	1	
Constrained V	0.019***	0.053***	0.050***	0.238***	1

Table 7: Cyclicality in turnover conditional on size bins and measures of financial constraints

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × GDP Growth	0.000	0.001	0.000	0.000	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
[99, 99.5] × GDP Growth	0.001	0.001	0.001	0.001	0.004	0.004
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
>99.5 × GDP Growth	-0.006	-0.005	-0.006	-0.006	-0.003	-0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Const. Adj. Eff. × GDP Growth		0.311***				
		(0.0564)				
Const. Overdue × GDP Growth			1.495***			
			(0.175)			
Const. Overdue Inc. × GDP Growth				0.882***		
				(0.217)		
Const. Maturing × GDP Growth					0.0855	
					(0.103)	
Const. Secured × GDP Growth						-0.145
						(0.102)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × GDP Growth FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1322120	1322120	1322120	1322120	1082432	1082432

Notes. Estimates report the semi-elasticity of turnover with respect to GDP. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 8: Elasticity of turnover to idiosyncratic TFP shocks conditional on size bins and measures of financial constraints

	(1)	(2)	(3)	(4)	(5)	(6)
TFP shock	0.273***	0.268***	0.265***	0.267***	0.266***	0.266***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)
[90, 99] × TFP shock	0.096***	0.098***	0.100***	0.099***	0.089***	0.089***
	(0.011)	(0.011)	(0.011)	(0.011)	(0.013)	(0.013)
[99, 99.5] × TFP shock	0.014	0.017	0.019	0.018	0.059	0.057
	(0.029)	(0.029)	(0.029)	(0.029)	(0.036)	(0.036)
>99.5 × TFP shock	0.002	0.007	0.010	0.008	0.011	0.010
	(0.041)	(0.041)	(0.041)	(0.041)	(0.049)	(0.050)
Const. Adj. Eff. × TFP shock		0.016***				
		(0.004)				
Const. Overdue × TFP shock			0.076***			
			(0.009)			
Const. Overdue Inc. × TFP shock				0.075***		
				(0.010)		
Const. Maturing × TFP shock					0.068***	
					(0.007)	
Const. Secured × TFP shock						0.065***
						(0.008)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1011102	1011102	1011102	1011102	816841	816841

Notes. Estimates report the semi-elasticity of turnover with respect to idiosyncratic TFP shocks. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 9: Elasticity of turnover to idiosyncratic financial shocks conditional on size bins and measures of financial constraints

	(1)	(2)	(3)	(4)	(5)	(6)
Bank shock	0.015*	-0.001	0.002	0.007	0.022*	0.025**
	(0.007)	(0.008)	(0.007)	(0.007)	(0.009)	(0.009)
[90, 99] × Bank shock	0.078**	0.089**	0.085**	0.084**	0.069*	0.071*
	(0.028)	(0.029)	(0.028)	(0.028)	(0.033)	(0.034)
[99, 99.5] × Bank shock	-0.050	-0.037	-0.036	-0.040	0.151	0.152
	(0.170)	(0.170)	(0.169)	(0.169)	(0.198)	(0.198)
>99.5 × Bank shock	-0.147	-0.133	-0.133	-0.139	-0.028	-0.035
	(0.127)	(0.127)	(0.127)	(0.127)	(0.107)	(0.107)
Const. Adj. Eff. × Bank shock		0.0135				
		(0.013)				
Const. Overdue × Bank shock			0.155**			
			(0.049)			
Const. Overdue Inc. × Bank shock				0.128*		
				(0.057)		
Const. Maturing × Bank shock					0.179***	
					(0.041)	
Const. Secured × Bank shock						0.073*
						(0.035)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1196505	1196505	1196505	1196505	980796	980796

Notes. Estimates report the semi-elasticity of turnover with respect to financial shocks estimated with the Amiti & Weinstein (2018) methodology. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 10: Cyclicality in employees conditional on size bins and measures of financial constraints

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × GDP Growth	-0.001*	-0.001*	-0.001*	-0.001*	-0.002***	-0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
[99, 99.5] × GDP Growth	-0.000	0.000	0.000	0.000	0.000	0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
>99.5 × GDP Growth	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Const. Adj. Eff. × GDP Growth		0.074*				
		(0.033)				
Const. Overdue × GDP Growth			0.764***			
			(0.087)			
Const. Overdue Inc. × GDP Growth				0.455***		
				(0.108)		
Const. Maturing × GDP Growth					0.110*	
					(0.052)	
Const. Secured × GDP Growth						-0.011
						(0.053)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × GDP Growth FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1360304	1360304	1360304	1360304	1116621	1116621

Notes. Estimates report the semi-elasticity of employees with respect to GDP. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 11: Elasticity of employees to idiosyncratic TFP shocks conditional on size bins and financial constraints

	(1)	(2)	(3)	(4)	(5)	(6)
TFP shock	-0.053***	-0.055***	-0.056***	-0.055***	-0.060***	-0.059***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
[90, 99] × TFP shock	0.011***	0.012***	0.013***	0.012***	0.010**	0.010**
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
[99, 99.5] × TFP shock	0.035***	0.036***	0.036***	0.036***	0.031**	0.031**
	(0.009)	(0.009)	(0.009)	(0.009)	(0.010)	(0.010)
>99.5 × TFP shock	0.013	0.015	0.015	0.014	0.023	0.023
	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.012)
Const. Adj. Eff. × TFP shock		0.005***				
		(0.001)				
Const. Overdue × TFP shock			0.017***			
			(0.004)			
Const. Overdue Inc. × TFP shock				0.010*		
				(0.004)		
Const. Maturing × TFP shock					0.008**	
					(0.003)	
Const. Secured × TFP shock						0.003
						(0.003)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1014676	1014676	1014676	1014676	819792	819792

Notes. Estimates report the semi-elasticity of employees with respect to idiosyncratic TFP shocks. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 12: Elasticity of employees to idiosyncratic financial shocks conditional on size bins and measures of financial constraints

	(1)	(2)	(3)	(4)	(5)	(6)
Bank shockt	0.001	-0.004	-0.006	-0.003	0.011	0.014^{*}
	(0.005)	(0.005)	(0.004)	(0.004)	(0.006)	(0.006)
[90, 99] × Bank shock	0.036^{**}	0.039^{**}	0.039^{**}	0.039^{**}	0.023	0.023
	(0.014)	(0.014)	(0.013)	(0.013)	(0.016)	(0.016)
[99, 99.5] × Bank shock	-0.007	-0.004	-0.002	-0.003	0.034	0.035
	(0.068)	(0.068)	(0.068)	(0.068)	(0.078)	(0.078)
>99.5 × Bank shock	0.030	0.035	0.031	0.034	0.058	0.055
	(0.065)	(0.065)	(0.065)	(0.065)	(0.099)	(0.099)
Const. Adj. Eff. × Bank shock		-0.011				
		(0.009)				
Const. Overdue × Bank shock			0.092^{***}			
			(0.025)			
Const. Overdue Inc. × Bank shock				0.087^{**}		
				(0.031)		
Const. Maturing × Bank shock					0.046^{*}	
					(0.020)	
Const. Secured × Bank shock						-0.006
						(0.018)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1230781	1230781	1230781	1230781	1011230	1011230

Notes. Estimates report the semi-elasticity of employees with respect to financial shocks estimated with the Amiti & Weinstein (2018) methodology. Constrained measures are constructed as outlined in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

Table 13: Elasticity of turnover to GDP changes conditional on size bins and financial constraints excluding firm fixed effects

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × GDP Growth	0.000	0.001	0.001	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
[99, 99.5] × GDP Growth	-0.002	-0.002	-0.002	-0.002	-0.002	-0.001
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
>99.5 × GDP Growth	-0.005	-0.004	-0.004	-0.004	-0.005	-0.005
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Const. Adj. Eff. × GDP Growth		0.213^{***}				
		(0.054)				
Const. Overdue × GDP Growth			1.426^{***}			
			(0.166)			
Const. Overdue Inc. × GDP Growth				0.988^{***}		
				(0.205)		
Const. Maturing × GDP Growth					0.666^{***}	
					(0.098)	
Const. Secured × GDP Growth						0.310^{**}
						(0.096)
Industry × GDP Growth FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1326447	1326447	1326447	1326447	1088781	1088781

Notes. Estimates report the semi-elasticity of turnover with respect to GDP changes. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 14: Elasticity of turnover to idiosyncratic TFP shocks conditional on size bins and financial constraints excluding firm fixed effects

	(1)	(2)	(3)	(4)	(5)	(6)
TFP shock	0.109***	0.101***	0.100***	0.102***	0.100***	0.101***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
[90, 99] × TFP shock	0.057***	0.062***	0.061***	0.060***	0.040***	0.041***
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
[99, 99.5] × TFP shock	-0.023	-0.017	-0.018	-0.019	-0.012	-0.014
	(0.015)	(0.015)	(0.015)	(0.015)	(0.018)	(0.018)
>99.5 × TFP shock	-0.029*	-0.022	-0.021	-0.023	-0.033*	-0.037*
	(0.015)	(0.015)	(0.015)	(0.015)	(0.016)	(0.016)
Const. Adj. Eff. × TFP shock		0.027***				
		(0.003)				
Const. Overdue × TFP shock			0.124***			
			(0.007)			
Const. Overdue Inc. × TFP shock				0.133***		
				(0.009)		
Const. Maturing × TFP shock					0.078***	
					(0.005)	
Const. Secured × TFP shock						0.068***
						(0.005)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1018654	1018654	1018654	1018654	826395	826395

Notes. Estimates report the semi-elasticity of turnover with respect to idiosyncratic TFP shocks. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 15: Elasticity of turnover to idiosyncratic financial shocks conditional on size bins and measures of financial constraints excluding firm fixed effects

	(1)	(2)	(3)	(4)	(5)	(6)
Bank shock	-0.036***	-0.074***	-0.056***	-0.051***	-0.020**	-0.028***
	(0.006)	(0.007)	(0.006)	(0.006)	(0.007)	(0.007)
[90, 99] × Bank shock	0.073**	0.102***	0.083***	0.083***	0.103***	0.110***
	(0.025)	(0.025)	(0.025)	(0.025)	(0.031)	(0.031)
[99, 99.5] × Bank shock	-0.054	-0.021	-0.040	-0.039	0.043	0.051
	(0.163)	(0.163)	(0.162)	(0.163)	(0.165)	(0.165)
>99.5 × Bank shock	-0.038	-0.004	-0.021	-0.025	0.012	0.015
	(0.112)	(0.112)	(0.112)	(0.112)	(0.105)	(0.105)
Const. Adj. Eff. × Bank shock		0.019				
		(0.012)				
Const. Overdue × Bank shock			0.085*			
			(0.039)			
Const. Overdue Inc. × Bank shock				0.081		
				(0.047)		
Const. Maturing × Bank shock					0.209***	
					(0.036)	
Const. Secured × Bank shock						0.089**
						(0.031)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1202889	1202889	1202889	1202889	989386	989386

Notes. Estimates report the semi-elasticity of turnover with respect to financial shocks estimated with the Amiti & Weinstein (2018) methodology. Constrained measures are constructed as outlined in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

Table 16: Cyclicality in turnover conditional on size bins and measures of financial constraints including time fixed effects

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × GDP Growth	0.000	0.001	0.001	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
[99, 99.5] × GDP Growth	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
>99.5 × GDP Growth	-0.005	-0.004	-0.004	-0.004	-0.005	-0.005
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Const. Adj. Eff. × GDP Growth		0.213^{***}				
		(0.054)				
Const. Overdue × GDP Growth			1.426^{***}			
			(0.166)			
Const. Overdue Inc. × GDP Growth				0.988^{***}		
				(0.205)		
Const. Maturing × GDP Growth					0.666^{***}	
					(0.098)	
Const. Secured × GDP Growth						0.310^{**}
						(0.096)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1326447	1326447	1326447	1326447	1088781	1088781

Notes. Estimates report the semi-elasticity of turnover with respect to GDP.Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 17: Cyclicality in turnover conditional on size bins and measures of financial constraints excluding firms that have 0 potential credit in all periods

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × GDP Growth	0.001	0.001	0.001	0.001	0.001	0.002
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
[99, 99.5] × GDP Growth	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002
	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)
>99.5 × GDP Growth	-0.004	-0.004	-0.004	-0.004	-0.004	-0.003
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Const. Adj. Eff. × GDP Growth		0.119				
		(0.069)				
Const. Overdue × GDP Growth			1.760^{***}			
			(0.212)			
Const. Overdue Inc. × GDP Growth				1.251^{***}		
				(0.265)		
Const. Maturing × GDP Growth					0.706^{***}	
					(0.102)	
Const. Secured × GDP Growth						0.363^{***}
						(0.103)
Industry × GDP Growth FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1161130	1161130	1161130	1161130	955844	955844

Notes. Estimates report the semi-elasticity of turnover with respect to GDP. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 18: Elasticity of turnover to idiosyncratic TFP shocks conditional on size bins and measures of financial constraints excluding firms that have 0 potential credit in all periods

	(1)	(2)	(3)	(4)	(5)	(6)
TFP shock	0.108***	0.101***	0.100***	0.102***	0.098***	0.099***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
[90, 99] × TFP shock	0.056***	0.059***	0.059***	0.058***	0.039***	0.040***
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
[99, 99.5] × TFP shock	-0.020	-0.015	-0.017	-0.017	-0.009	-0.011
	(0.015)	(0.015)	(0.015)	(0.015)	(0.017)	(0.018)
>99.5 × TFP shock	-0.024	-0.018	-0.017	-0.019	-0.027	-0.032
	(0.015)	(0.015)	(0.015)	(0.015)	(0.016)	(0.016)
Const. Adj. Eff. × TFP shock		0.034***				
		(0.003)				
Const. Overdue × TFP shock			0.137***			
			(0.009)			
Const. Overdue Inc. × TFP shock				0.151***		
				(0.011)		
Const. Maturing × TFP shock					0.078***	
					(0.006)	
Const. Secured × TFP shock						0.069***
						(0.005)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	907128	907128	907128	907128	738944	738944

Notes. Estimates report the semi-elasticity of turnover with respect to idiosyncratic TFP shocks. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 19: Elasticity of turnover to idiosyncratic financial shocks conditional on size bins and measures of financial constraints excluding firms that have 0 potential credit in all periods

	(1)	(2)	(3)	(4)	(5)	(6)
Bank shock	-0.047***	-0.070***	-0.063***	-0.058***	-0.030***	-0.038***
	(0.006)	(0.007)	(0.006)	(0.006)	(0.008)	(0.008)
[90, 99] × Bank shock	0.076**	0.093***	0.085***	0.084***	0.102***	0.108***
	(0.025)	(0.025)	(0.025)	(0.025)	(0.031)	(0.031)
[99, 99.5] × Bank shock	-0.101	-0.082	-0.090	-0.090	-0.013	-0.006
	(0.163)	(0.163)	(0.163)	(0.163)	(0.167)	(0.167)
>99.5 × Bank shock	-0.073	-0.052	-0.059	-0.064	-0.012	-0.010
	(0.117)	(0.117)	(0.117)	(0.117)	(0.105)	(0.106)
Const. Adj. Eff. × Bank shock		0.005				
		(0.015)				
Const. Overdue × Bank shock			0.132*			
			(0.058)			
Const. Overdue Inc. × Bank shock				0.106		
				(0.071)		
Const. Maturing × Bank shock					0.212***	
					(0.039)	
Const. Secured × Bank shock						0.105**
						(0.034)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
N	1071731	1071731	1071731	1071731	883661	883661

Notes. Estimates report the semi-elasticity of turnover with respect to financial shocks estimated with the Amiti & Weinstein (2018) methodology. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 20: Cyclicality in turnover conditional on size bins and measures of financial constraints including bank controls

	(1)	(2)	(3)	(4)	(5)	(6)
[90, 99] × Δ GDP	-0.003	-0.003	-0.002	-0.002	-0.004	-0.004
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
[99, 99.5] × Δ GDP	-0.002	-0.002	-0.001	-0.001	-0.007	-0.007
	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)
>99.5 × Δ GDP	0.002	0.002	0.004	0.005	-0.008	-0.008
	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
Const. Adj. Eff. × Δ GDP		0.205^{**}				
		(0.0641)				
Const. Overdue × Δ GDP			1.955^{***}			
			(0.483)			
Const. Overdue Inc. × Δ GDP				1.706^{**}		
				(0.594)		
Const. Maturing × Δ GDP					1.036^{***}	
					(0.281)	
Const. Secured × Δ GDP						0.502
						(0.288)
Industry × Δ GDP FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
Bank Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	1187112	1187112	1187112	1187112	976408	976408

Notes. Estimates report the semi-elasticity of turnover with respect to GDP. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 21: Elasticity of turnover to idiosyncratic TFP shocks conditional on size bins and measures of financial constraints including bank controls

	(1)	(2)	(3)	(4)	(5)	(6)
TFP shock	0.112***	0.104***	0.103***	0.105***	0.101***	0.103***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
[90, 99] × TFP shock	0.056***	0.060***	0.059***	0.059***	0.039***	0.040***
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
[99, 99.5] × TFP shock	-0.023	-0.017	-0.019	-0.019	-0.011	-0.013
	(0.015)	(0.015)	(0.015)	(0.015)	(0.018)	(0.018)
>99.5 × TFP shock	-0.027	-0.020	-0.020	-0.020	-0.029	-0.034*
	(0.015)	(0.015)	(0.015)	(0.015)	(0.016)	(0.017)
Const. Adj. Eff. × TFP shock		0.033***				
		(0.003)				
Const. Overdue × TFP shock			0.125***			
			(0.008)			
Const. Overdue Inc. × TFP shock				0.136***		
				(0.010)		
Const. Maturing × TFP shock					0.078***	
					(0.005)	
Const. Secured × TFP shock						0.070***
						(0.005)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
Bank Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	892702	892702	892702	892702	725166	725166

Notes. Estimates report the semi-elasticity of turnover with respect to idiosyncratic TFP shocks. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 22: Elasticity of turnover to idiosyncratic financial shocks conditional on size bins and measures of financial constraints including bank controls

	(1)	(2)	(3)	(4)	(5)	(6)
Bank shock	-0.025***	-0.050***	-0.041***	-0.039***	-0.020*	-0.025**
	(0.007)	(0.008)	(0.007)	(0.007)	(0.009)	(0.009)
[90, 99] × Bank shock	0.067*	0.082**	0.072*	0.074*	0.122**	0.127***
	(0.029)	(0.029)	(0.029)	(0.029)	(0.038)	(0.038)
[99, 99.5] × Bank shock	-0.106	-0.086	-0.096	-0.093	-0.010	-0.004
	(0.168)	(0.168)	(0.168)	(0.168)	(0.171)	(0.171)
>99.5 × Bank shock	-0.125	-0.102	-0.108	-0.111	-0.043	-0.051
	(0.184)	(0.184)	(0.184)	(0.184)	(0.249)	(0.248)
Const. Adj. Eff. × Bank shock		0.061***				
		(0.016)				
Const. Overdue × Bank shock			0.146**			
			(0.049)			
Const. Overdue Inc. × Bank shock				0.169**		
				(0.061)		
Const. Maturing × Bank shock					0.216***	
					(0.038)	
Const. Secured × Bank shock						0.091**
						(0.033)
Industry × Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
Bank Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	1153335	1153335	1153335	1153335	947734	947734

Notes. Estimates report the semi-elasticity of turnover with respect to financial shocks estimated with the Amiti & Weinstein (2018) methodology. Constrained measures are constructed as documented in Section 2.1. All specifications contain a constant term and non-interacted indicators. Standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 23: Parameter values benchmark calibration

Parameter	Description	Value	Source
β	Discount factor	0.96	K&T (2013)
α	Returns on capital	0.30	K&T (2013)
η	Returns on labor	0.60	K&T (2013)
δ	Depreciation rate	0.065	K&T (2013)
ψ	Labour preference	2.15	K&T (2013)
π_d	Exogenous probability of exit	0.02	Data
μ_θ	Average: permanent productivity	0	Normalized
μ_w	Average: transitory shock	0	Normalized
Model			
ξ	Collateral constraint	0.50	Calibrated
σ_θ	Std. dev.: permanent productivity	0.20	Calibrated
ρ_w	Persistence of transitory shock	0.43	Calibrated
σ_w	Std. dev: transitory shock	0.11	Calibrated
μ_{ke}	Relative size of entrants	0.01	Calibrated
σ_{ke}	Standard deviation of entrants	0.35	Calibrated

Notes. K&T (2013) is short for Khan & Thomas (2013).

Table 24: Parameter values with calibration directly targeting size-constraint moments

Parameter	Description	Value	Source
ξ	Collateral constraint	0.77	Calibrated
σ_θ	Std. dev.: permanent productivity	0.22	Calibrated
ρ_w	Persistence of transitory shock	0.11	Calibrated
σ_w	Std. dev: transitory shock	0.84	Calibrated
μ_{ke}	Relative size of entrants	0.09	Calibrated
σ_{ke}	Standard deviation of entrants	0.19	Calibrated

Table 25: Targeted size-constraint moments

Moment	Data	Model
Percentage of const. firms	0.23	0.24
Share of const. firms in bottom 20%	0.33	0.31
Size of 90th percentile / median	9.44	12.18
Size of const. firms 90th percentile / median	7.35	5.99
Size of unconst. firms 90th percentile / median	9.67	9.24
Asset share of const. firms	0.07	0.16
Share of const. firms in top 10% vs. bottom 20%	0.36	0.19
Percentage of const. firms in top 1%	0.09	0.03

Notes. All constrained firms moments are calculated using constrained measure I.

Table 26: Calibration fit for a model with no permanent productivity component

Moment	Data	Model
Percentage of const. firms	0.23	0.08
Share of const. firms in bottom 20%	0.33	0.37
Size of 90th-percentile vs. median	9.44	9.35
Size of const. firms 90th-percentile vs. median	7.35	2.12
Size of unconst. firms 90th-percentile vs. median	9.67	7.65
Asset share of const. firms	0.07	0.05
Share of const. firms in top 10% vs. bottom 20%	0.36	0
Percentage of const. firms in top 1%	0.09	0

Notes. All constrained firms moments are calculated using constrained measure I

C Additional figures

C.1 Descriptive figures

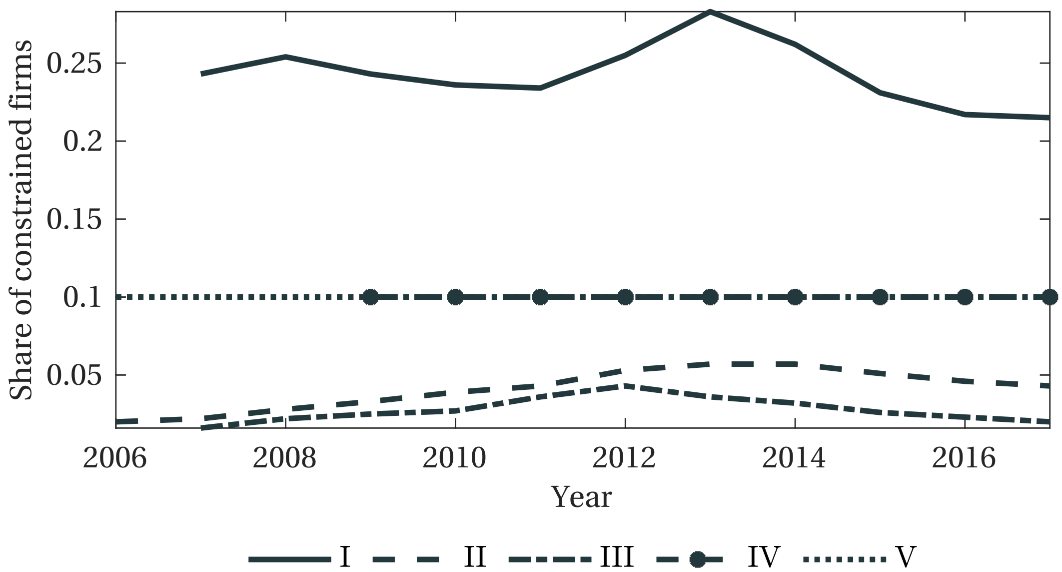


Figure 7: Share of constrained firms over time. Measures 1 to 5 as defined in Section 2.1

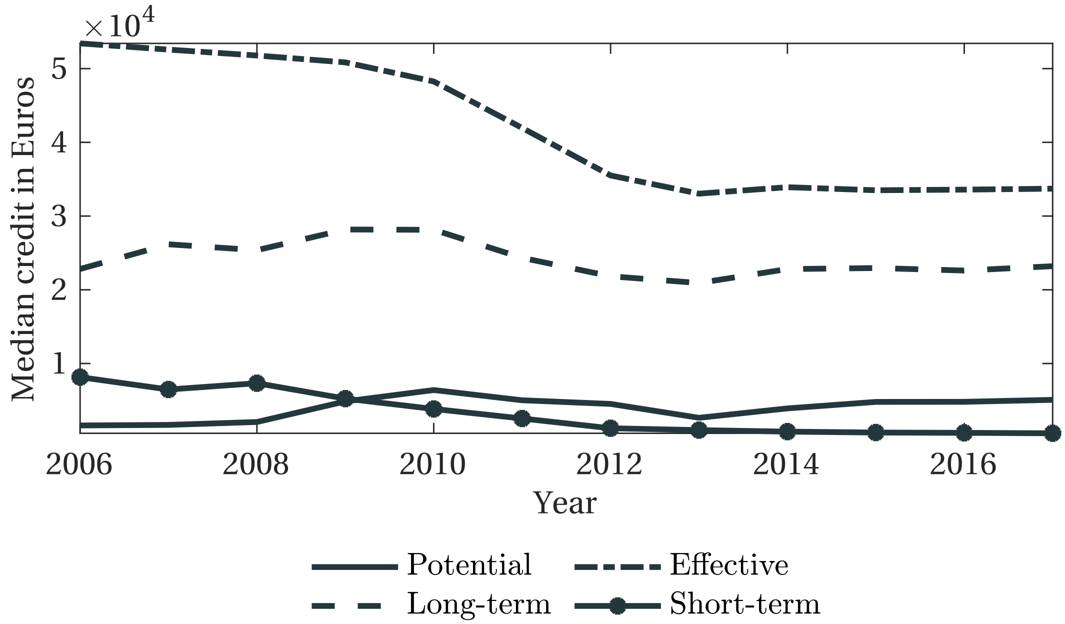


Figure 8: Median values for potential, effective, long-term and short-term credit over time.

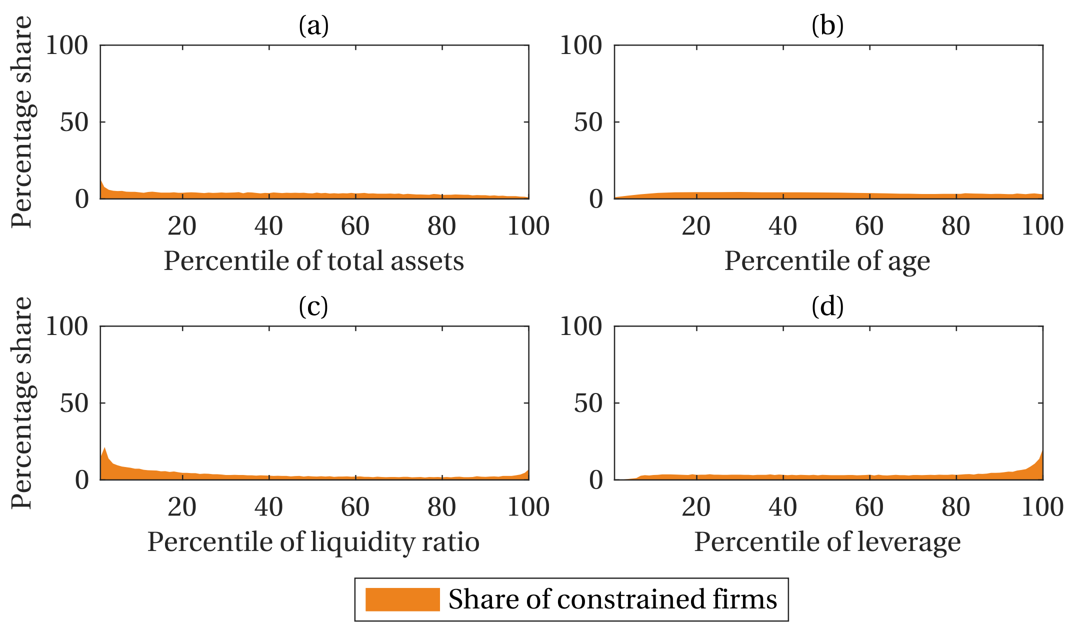


Figure 9: Decomposition of constrained and unconstrained firms across percentiles of firm variables using constraint measure II

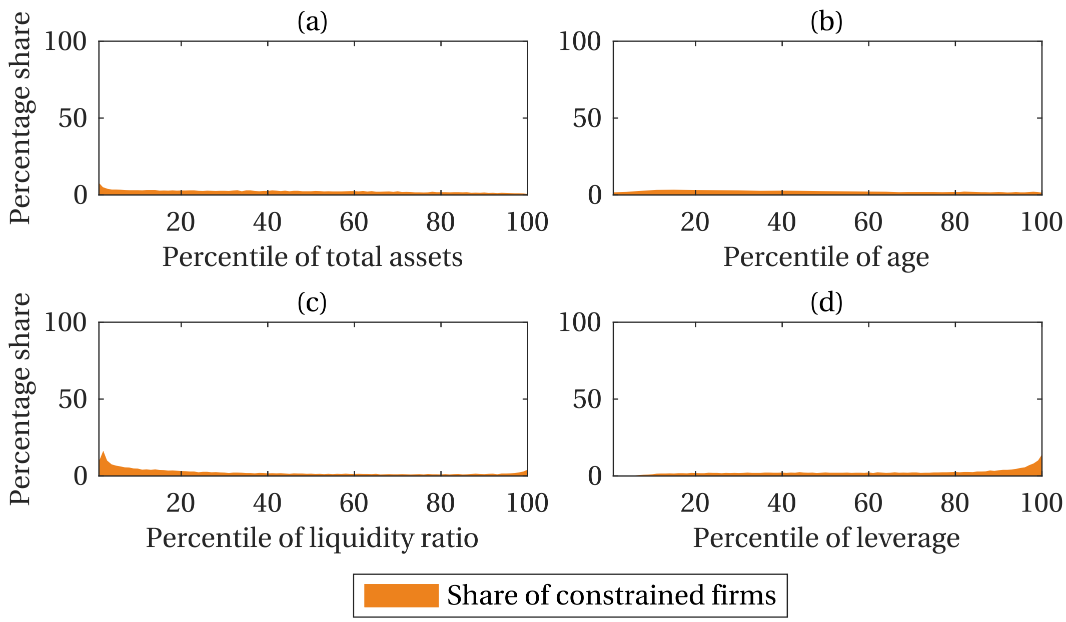


Figure 10: Decomposition of constrained and unconstrained firms across percentiles of firm variables using constraint measure III

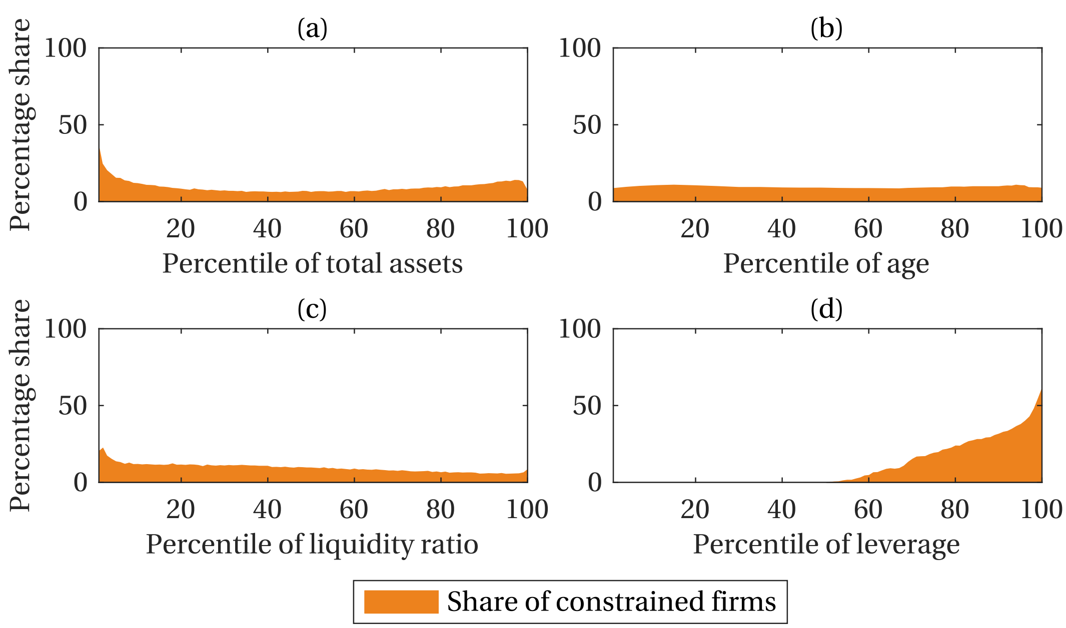


Figure 11: Decomposition of constrained and unconstrained firms across percentiles of firm variables using constraint measure IV

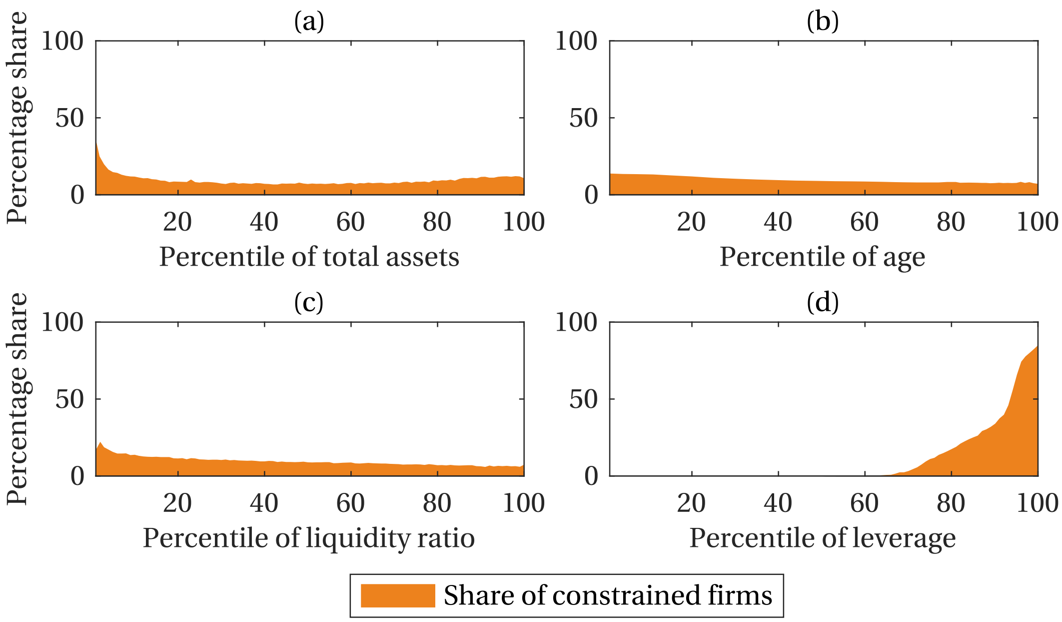


Figure 12: Decomposition of constrained and unconstrained firms across percentiles of firm variables using constraint measure V

C.2 Statistical model

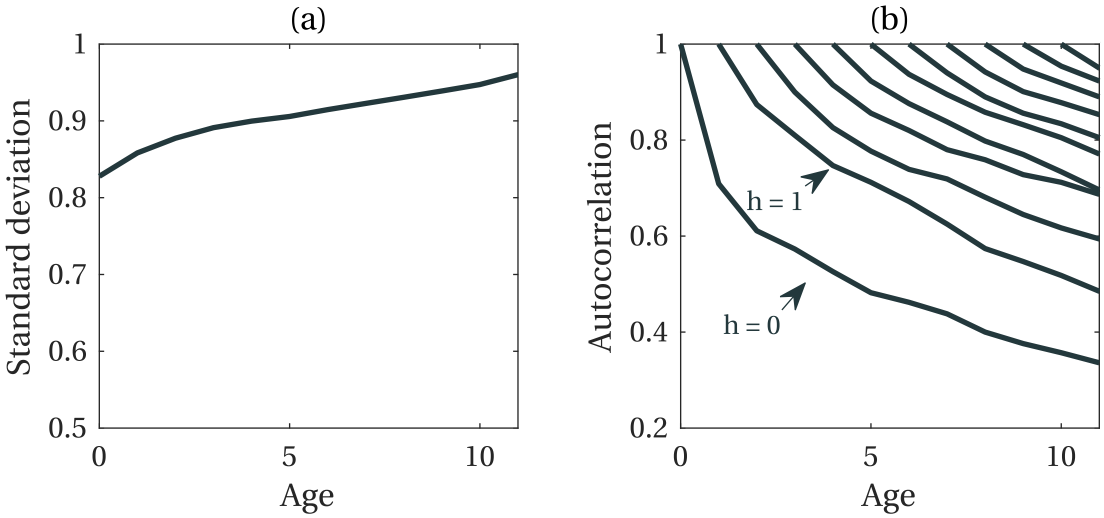


Figure 13: Standard deviation and autocorrelation of log employment by age. The left panel presents the standard deviation of log employment by age, after controlling for sector and year fixed effects. The right panel presents the autocorrelation of log employment between ages a and $h \leq a$. Across lines h changes, while a changes along the lines

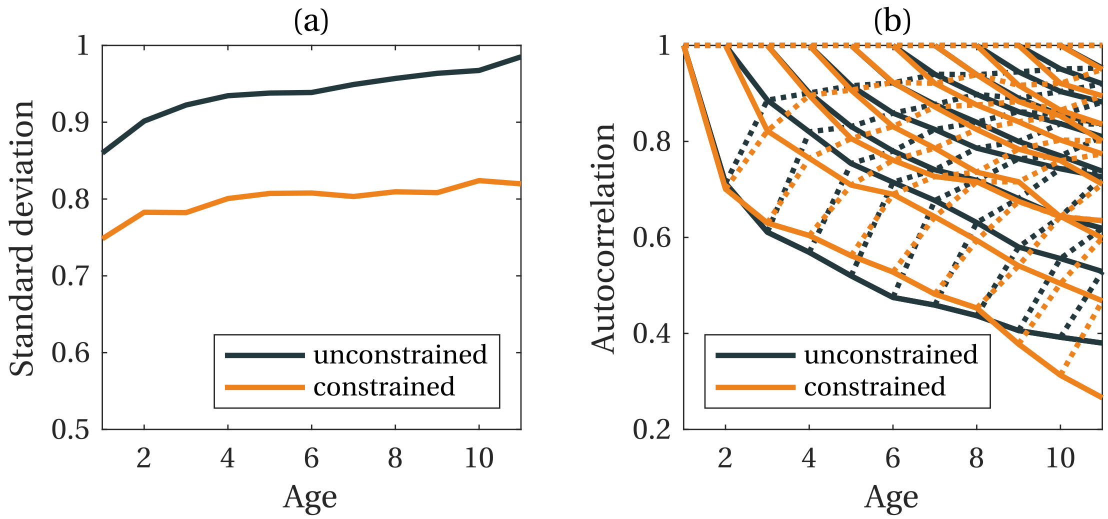


Figure 14: Standard deviation and autocorrelation of log employment by age and separated by constraint measure I. The left panel presents the standard deviation of log employment by age, after controlling for sector and year fixed effects. The right panel presents the autocorrelation of log employment between ages a and $h \leq a$. Across lines h changes, while a changes along the lines

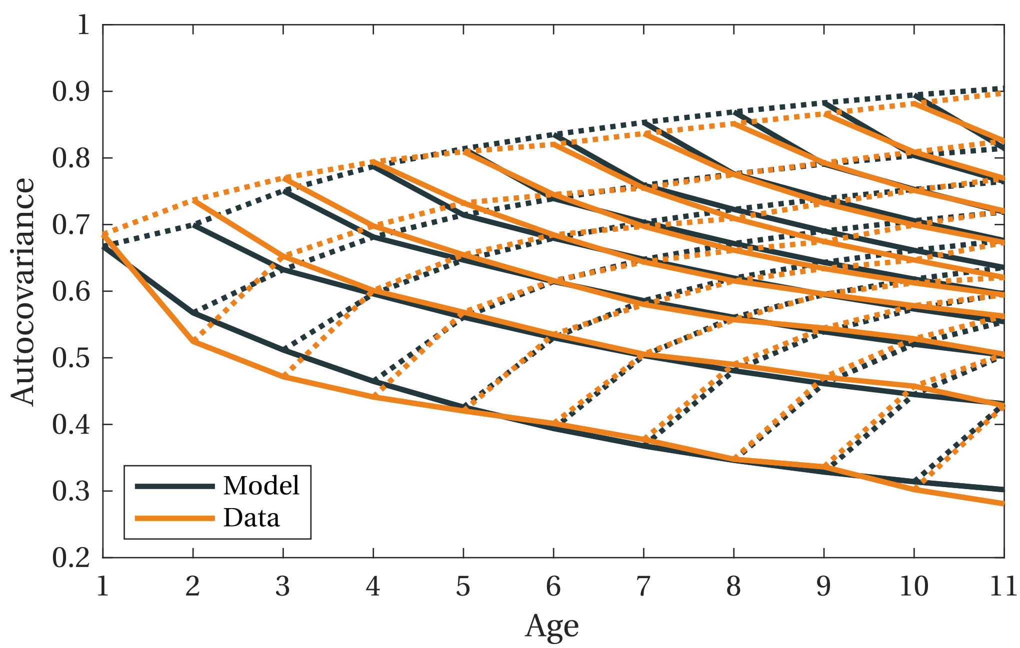


Figure 15: Model fit of statistical model for employment process

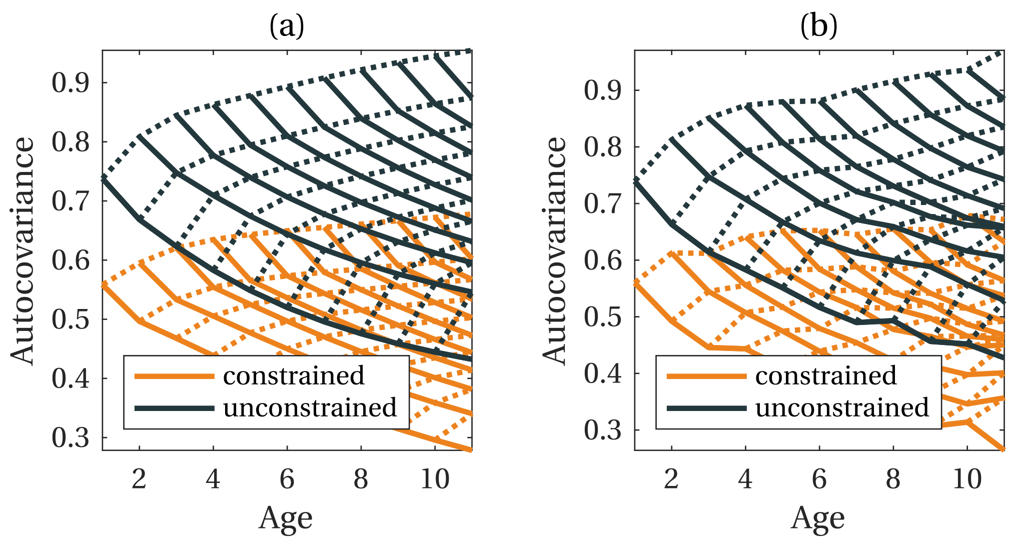
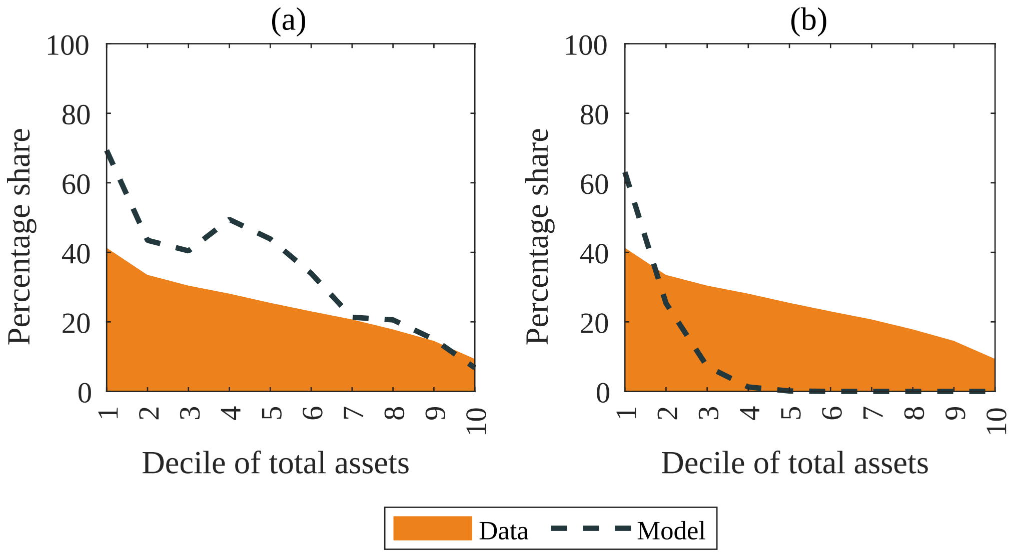


Figure 16: Empirical and model autocovariance for constrained firms (orange) and unconstrained firms (blue) using the measure Constrained I

C.3 Quantitative model



(a) Transitory shock

Figure 17: Share of constrained firms across the size distribution. On the left panel the benchmark model when calibrated directly to target the joint size-constraint distribution. On the right panel the model when shutting down the permanent productivity component. Calibration results for the two models are presented in Tables 25 and 26.

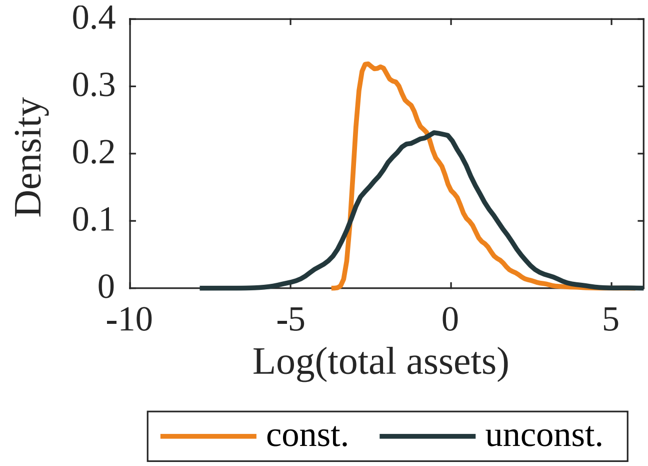


Figure 18: Conditional distributions of log of total assets implied by the model

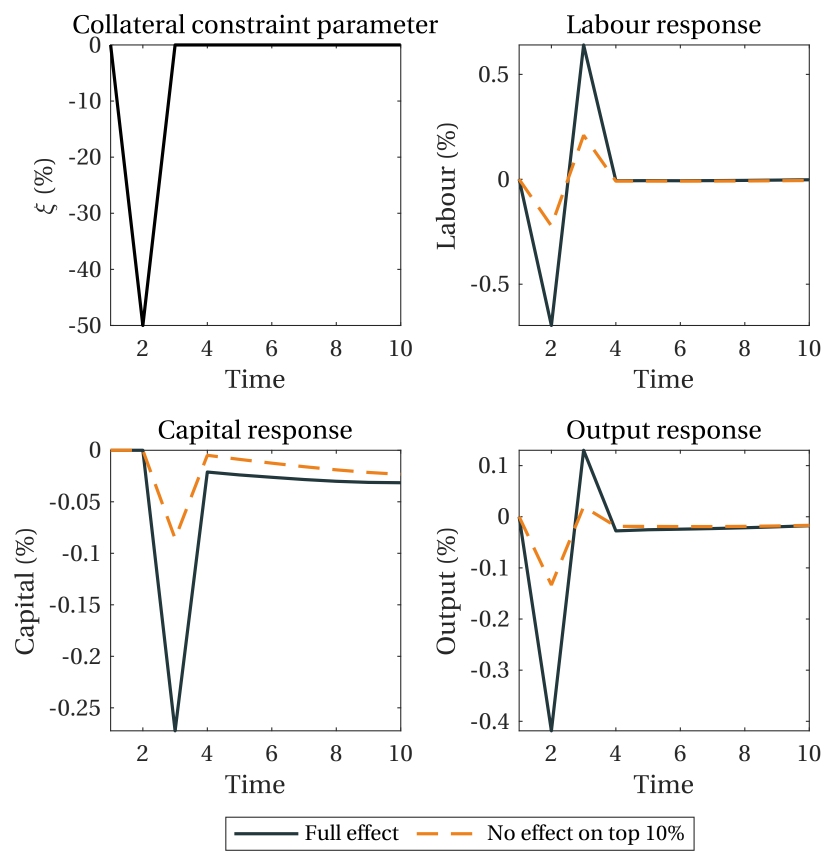


Figure 19: General equilibrium IRFs to a financial shock. Lines indicate the partial equilibrium response to a shock to ξ in the upper left panel.

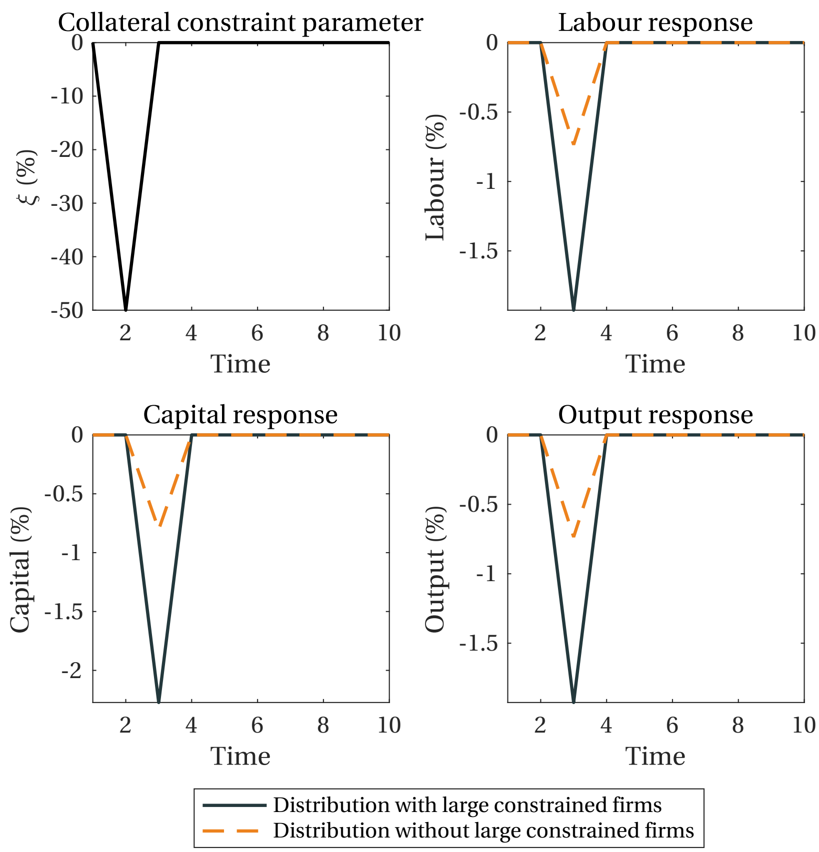


Figure 20: IRFs to a financial shock. Lines indicate the partial equilibrium response to a shock to ξ in the upper left panel, with wages fixed at their steady state level.

D Firm potential

In contrast to our stylised facts, a firm dynamics financial frictions model with only a transitory productivity shock à la Khan & Thomas (2013) predicts a very strong correlation between firm size and financial constraints, as firms require a relatively uniform minimum size to become unconstrained. One factor that could potentially break this strong correlation is heterogeneous ex-ante conditions for firms, such as firm potential. Small firms may be unconstrained as they already have reached their potential - i.e. optimal size - while large firms may still be growing and are still constrained. Equally, heterogeneous potential creates a dispersion of unconstrained firms across the entire firm size distribution, similar to our first stylised fact. Further, larger constrained firms may elevate the fraction of assets held by constrained firms closer to what we observe in the data. Finally, this heterogeneity may also explain why financial factors matter for firm cyclicality even when controlling for firm size, as demonstrated in our third stylised fact. Accordingly, this section investigates whether such ex-ante heterogeneity exists in our dataset.

Looking at the standard deviations of log employment by age a and autocorrelation structure of log employment between age a and h, we find evidence that there is ex-ante heterogeneity, as firms at birth are not all equal, suggesting that ex-ante conditions are persistent and affect firms even in the long run, in line with evidence presented by Pugsley et al. (2021).[18]

Additionally, we find evidence that the ex-ante heterogeneity affects constrained and unconstrained firms differently.[19] The standard deviation is lower throughout the life-cycle and the autocorrelation structure converges to a lower level for constrained firms compared to unconstrained ones. One may have expected the opposite to be true, as constrained firms potentially have less resources to grow and so their employment tomorrow could have a stronger corre-

[18]To prevent differences across sectors and business cycle conditions from explaining the majority of the standard deviation and autocorrelation, we first control for sector and year fixed effects and then use the residuals of log employment.

[19]Here we are using the baseline measure Const I, taking into account both potential credit and growth of effective credit. A firm is considered constrained if at age $a-h$ it has potential credit equal to zero and if the effective credit is not growing.

lation with employment today. Yet, the fact that the autocorrelation tends to be higher across the life-cycle for unconstrained firms may be indicative that they are born closer to their optimal size, when compared to constrained firms. This may then explain why some young firms are constrained and others are not: the ones born closer to their optimal size have lower investments and do not become constrained, while firms that need to grow to reach the optimal size exhaust their credit lines. The results are depicted in Figures 13 and 14 in Appendix C.

Statistical model. To gain understanding beyond descriptive statistics of the importance of ex-ante and ex-post heterogeneity for the life-cycle of firms, we again follow Pugsley et al. (2021) and adopt their statistical model. This model uses the information provided by the autocovariance structure of log employment to capture the importance of both types of heterogeneity.

Consider the following decomposition for employment n by firm i at age a:

$$\underbrace{\ln n_{i,a}}_{\text{log employment}} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante component}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post component}}, \tag{3}$$

where

$$\begin{array}{llll}
u_{i,a} = \rho_u u_{i,a-1} + \theta_i, & u_{i,-1} \sim iid\left(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}\right), & \theta_i \sim iid\left(\mu_\theta, \sigma^2_\theta\right), & \left|\rho_u\right| \leq 1 \\
v_{i,a} = \rho_v v_{i,a-1}, & v_{i,-1} \sim iid\left(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}\right), & & \left|\rho_v\right| \leq 1 \\
w_{i,a} = \rho_w w_{i,a-1} + \varepsilon_{i,a}, & w_{i,-1} = 0, & \varepsilon_{i,a} \sim iid\left(0, \sigma^2_\varepsilon\right), & \left|\rho_w\right| \leq 1 \\
z_{i,a} \sim iid\left(0, \sigma^2_z\right) & & &
\end{array}$$

In this employment process, the terms $u_{i,a}$ and $v_{i,a}$ capture the ex-ante profile while $w_{i,a}$ and $z_{i,a}$ capture the ex-post one. The ex-ante component is determined by three shocks that are drawn just prior to the birth year, at $a = -1$. The shocks $v_{i,-1}$ and $u_{i,-1}$ represent the initial conditions of the firm, which allow for rich heterogeneity even at birth. θ_i is the permanent component, which will accumulate over the life-cycle at speed ρ_u. In particular, with $\rho_u < 1$, the long-run steady state level of employment will be given by $\frac{\theta_i}{1-\rho_u}$. Further, this specification allows for rich heterogeneity not only in terms of optimal size of the firms, de-

Table 27: Calibrated model parameters for the unbalanced panel, including all, constrained and unconstrained firms according to measure I

	ρ_u	ρ_v	ρ_w	σ_θ	σ_u	σ_v	σ_ϵ	σ_z
Total	0.425	0.799	0.904	0.369	0.748	0.708	0.305	0.185
Unconstrained	0.431	0.770	0.884	0.399	0.769	0.744	0.311	0.158
Constrained	0.493	0.874	0.911	0.255	0.655	0.641	0.265	0.176

pending on the distribution of θ_i, but also in terms of the speed at which firms reach the steady state. As firms start at different points depending on $u_{i,-1}$ and $v_{i,-1}$ and each shock has its own persistence parameter, the path from initial to steady state employment will highly differ across firms.

The ex-post component is formed of two different shocks, one i.i.d. shock with expected value of zero, and a persistent one that follows an AR(1) process with i.i.d. innovations $\epsilon_{i,a}$ and persistence ρ_w. To abstract the ex-post component from affecting the ex-ante one, we set the initial conditions of the persistent shock to $w_{i,-1} = 0$.

We calibrate the model for all, constrained and unconstrained firms separately by minimising the sum of squared differences between the model and empirical autocovariance. Firms are again split into constrained and unconstrained categories according to the measure Constrained I.

Table 27 presents the parameters resulting from the calibration strategy.[20] Two key parameters of the model are ρ_u and σ_θ, as, together, they imply that permanent heterogeneity exists. First, using the total panel, the point estimates imply that ex-ante conditions matter, as both ρ_u and σ_θ are nonzero. Second, the point estimates imply a standard deviation of steady state employment, σ_θ for unconstrained firms of 0.399 and 0.255 for constrained ones. This again demonstrates that there seem to be differences between both types of firms that originate from ex-ante conditions.

Finally, to more clearly identify the ex-post and ex-ante contributions, one

[20]Figure 15 in Appendix C.2 plots the model fit to the data for when calibrate to all firms in the Portuguese economy. Figure 16 in Appendix C.2 presents the fit when the model is calibrated for constrained and unconstrained firms separately.

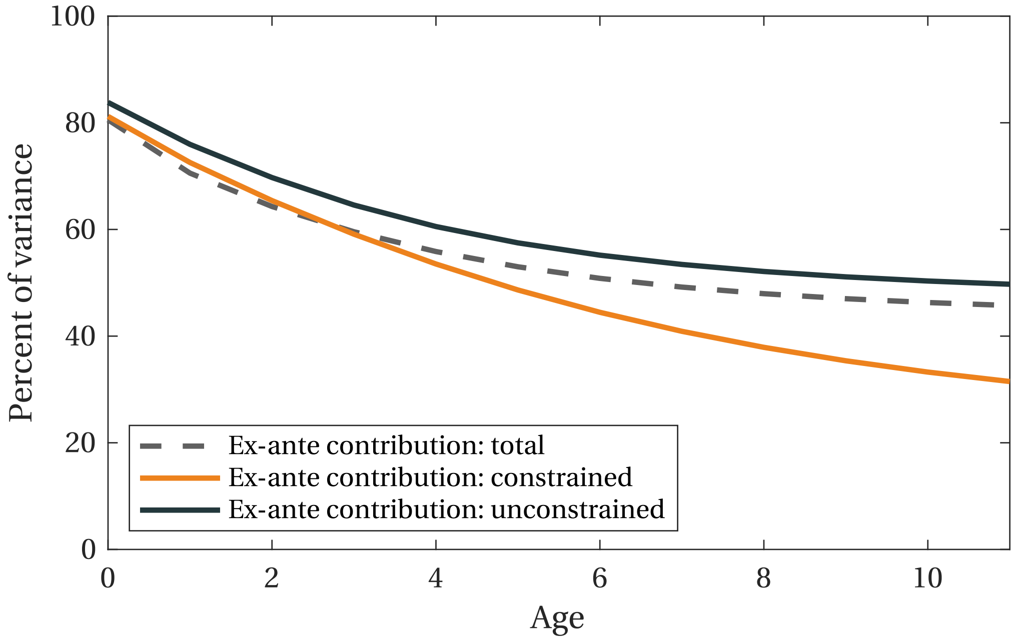


Figure 21: Ex-ante variance contribution. Values for constrained firms presented in orange (light), while blue stands for the unconstrained firms (dark).

can also derive the formula for the model autocovariance, enabling a clear identification of the contribution of both components. The autocovariance formula is given by

$$Cov[\ln n_{i,a}, \ln n_{i,a-j}] = \underbrace{\left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right)\sigma_\theta^2 + \rho_u^{2(a+1)-j}\sigma_{\hat{u}}^2 + \rho_v^{2(a+1)-j}\sigma_{\hat{v}}^2}_{\text{Ex-ante component}}$$

$$\underbrace{+\,\sigma_\epsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \sigma_z^2 \mathbf{1}_{j=0}}_{\text{Ex-post component}}$$

and its derivation can be found in Appendix E. The autocovariance is a function of variance and persistence parameters of both ex-ante and ex-post shocks, as described above. Figure 21 illustrates the importance of the ex-ante component for the variance as a function of a firm's age. For all categories of firms, the ex-ante component contribution is above 80% at birth. Differences between the constrained and unconstrained firms start to arise after year 1, with the ex-ante component explaining more than 60% of the variance for unconstrained firms in the long run, while for constrained firms it is below 40%.

The fact that the ex-ante contribution is stronger for unconstrained firms is indicative that these firms are born closer to their optimal size. At the same time, constrained firms have not reached their optimal size yet, and so naturally less contribution to the employment dispersion originates from permanent conditions.

All the empirical evidence in this section suggests that ex-ante heterogeneity: 1) matters both in the short and in the long-run and 2) more strongly affects unconstrained than constrained firms. This may be indicative that unconstrained firms start closer to their steady state level of employment, while firms that still need to grow exhaust their credit lines to reach their optimal size and so become constrained. This mechanism is mirrored in our general equilibrium firm dynamics model in the next section.

E Statistical model derivation

This is reproduced from Pugsley et al. (2021) for reference. Write stochastic processes in MA representation:

$$
\begin{aligned}
u_{i,t} &= \rho_u^{t+1} u_{i,-1} + \sum_{k=0}^{a} \rho_u^k \theta_i \\
v_{i,a} &= \rho_v^{a+1} v_{i,-1} \\
w_{i,a} &= \sum_{k=0}^{a} \rho_w^k \varepsilon_{i,a-k} = \sum_{k=0}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} \quad 0 \leq j \leq a
\end{aligned}
$$

So the level of log employment of firm i at age a is:

$$
\ln n_{i,a} = \rho_u^{a+1} u_{i,-1} + \sum^{a} \rho_u^k \theta_i + \rho_v^{a+1} v_{i,-1} + \sum_{i=1}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{i=1}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} + z_{i,a}
$$

Then the autocovariance of log employment at age a and $a-j$ for $j \geq 0$ is:

$$
\begin{aligned}
\operatorname{Cov}\left[\log n_{i,a}, \log n_{i,a-j}\right] &= \left(\sum_{k=0}^{a} \rho_u^k\right) \sigma_\theta^2 \left(\sum_{k=0}^{a-j} \rho_u^k\right) + \rho_u^{a+1} \sigma_{\tilde{u}}^2 \rho_u^{a-j+1} + \rho_v^{a+1} \sigma_{\tilde{v}}^2 \rho_v^{a-j+1} \\
&\quad + \operatorname{Cov}\left[\rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}, \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}\right] + \mathbf{1}_{\{j=0\}} \sigma_z^2 \\
&= \sigma_\theta^2 \left(\sum_{k=0}^{a} \rho_u^k\right) \left(\sum_{k=0}^{a-j} \rho_u^k\right) + \sigma_{\tilde{u}}^2 \rho_u^{2(a+1)-j} + \sigma_{\tilde{v}}^2 \rho_v^{2(a+1)-j} + \sigma_\varepsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \mathbf{1}_{\{j=0\}} \sigma_z^2
\end{aligned}
$$

F Model: Firm level decisions

Unconstrained Firms This group of firms can implement both the optimal amount of capital and the minimum savings policy that guarantees these firms will never be constrained in the future again. Given the absence of adjustment costs and the stochastic process for φ the optimal amount of capital is the solution to:

$$\max_{k'} \quad -k' + \beta \mathbb{E}_{\varphi'|\varphi}\left[(\pi(k',\varphi') + (1-\delta)k'\right]$$

So the optimal amount of capital solves the following equation

$$\beta \mathbb{E}_{\varphi'|\varphi}\left[\frac{\partial \pi}{\partial k'}(k',\varphi')\right] = 1 + \beta\delta - \beta$$

which is when the expected marginal productivity of capital is equal to the marginal cost of an extra unit. The minimum savings policy these firms implement guarantees they will never be constrained again. It is given by

$$B^*(\varphi_i) = \min_{\varphi_j} \tilde{B}(k^*(\varphi_i), \varphi_j)$$

where $\tilde{B}(k^*(\varphi_i), \varphi_j)$ is the minimum savings that guarantees that going from state φ_i to φ_j the firm is still able to implement the optimal amount of capital. It is given by

$$\begin{aligned}\tilde{B}(k^*(\varphi_i), \varphi_j) =& \pi(k^*(\varphi_i), \varphi_j) + (1-\delta)k^*(\varphi_i) - k'^*(\varphi_j) + \\ & q \min\left\{B^*(\varphi_j), \xi\left(\pi(k^*(\varphi_i), \varphi_j) + (1-\delta)k^*(\varphi_i) - \tilde{B}(k^*(\varphi_i), \varphi_j)\right)\right\}\end{aligned}$$

Given the optimal amount of capital and the minimum savings policy, the dividends distributed by the unconstrained firms are given by

$$D = x - k^* + qB^*$$

From the dividend constraint $D \geq 0$ we can extract the minimum threshold for cash-on-hand that guarantees the firm is not constrained

$$\tilde{x} = k^* - qB^*$$

and the firms is constrained if $x \leq \tilde{x}$.

Constrained Firms: Type 1 These firms can implement the optimal amount of capital, k^*, but not the optimal savings policy and are therefore partially constrained. As they may still be constrained in future states, they value internal financing more than households value dividends. As a result, for this type of firms, $D = 0$. The amount of debt is given by

$$b' = \frac{(k^* - x)}{q}$$

A firm is type 1 if it can adopt the above amount of debt and capital and at the same time guaranteeing that it does not default in the next period.

Constrained Firms: Type 2 Strictly constrained firms can not implement the optimal amount of capital. Those firms utilize all their borrowing capacity as their marginal value of net worth is greater than unity. Hence, their savings policy is simply

$$b' = \xi x,$$

and their maximum possible investment is consequently

$$k' = x + q\xi x < k^*,$$

which is strictly smaller than their optimal level of capital k^*.

G Simple model: Results

Take a very simple model to analyze the impact of heterogeneous productivities on cyclicality, following Crouzet & Mehrotra (2020). Firms can only invest in physical capital, have permanent productivity and face no uncertainty, except for a stochastic death shock. The problem can be written as:

$$
\begin{aligned}
& V(k_{t,i}, b_{t,i}, \theta_i) = \pi_d x_{t,i} + (1-\pi_d)\left(x_{t,i} - k_{t+1,i} + q_t b_{t+1,i} + \beta V(k_{t+1,i}, b_{t+1,i}, \theta_i)\right) \\
& \text{subject to} \\
& x_{t,i} = z_t \theta_i k_{t,i}^{\alpha} + (1-\delta) k_{t,i} - b_{t,i} \\
& \xi x_{t,i} \geq b_{t+1,i} \\
& k_{t+1,i} \leq x_{t,i} + q_t b_{t+1,i}
\end{aligned}
$$

G.1 Unconstrained firms

Steady state growth. Unconstrained firms optimal capital $k^*_{t+1,i}$ is the solution to:

$$\beta^{-1} = (1-\delta) + \alpha z_t \theta_i k^{*\alpha-1}_{t+1,i}$$

Hence optimal capital $k^*_{t+1,i}$ is

$$k^*_{t+1,i} = \theta_i^{\frac{1}{1-\alpha}} \left(\frac{\alpha z_{t+1}}{\beta^{-1} - (1-\delta)}\right)^{\frac{1}{1-\alpha}}$$

where we can choose $z := \left(\frac{\beta^{-1}-(1-\delta)}{\alpha}\right)$ such that, at steady state and for $\theta = 1$, we have that $k^*_{t+1,i} = 1$. In the absence of idiosyncratic shocks and constant total factor productivity z, unconstrained firms are not growing at steady state as they reached their optimal level of capital.

$$g_{uncons} = \frac{k^*_{t+1,i}(\theta_i)}{k^*_{t,i}(\theta_i)} = 1$$

Cyclicality. Now consider the following setup; at time $t = -1$, $z_t = z$. At time $t = 0$, firms learn the future path of z_t, for $t \geq 0$ will be

$$z_t = z \exp(\rho^t \epsilon)$$

The growth rate then becomes

$$g_{uncons} = \frac{k^*_{t+1,i}(\theta_i)}{k^*_{t,i}(\theta_i)} = \frac{\exp(\frac{\rho}{1-\alpha}\epsilon)\theta_i^{1/(1-\alpha)}}{\theta_i^{1/(1-\alpha)}} = \exp\left(\frac{\rho}{1-\alpha}\epsilon\right)$$

Hence, the elasticity of capital is the same across all unconstrained firms, independent of firm size and firm-specific productivity.

$$\frac{\Delta g_{uncons}}{\Delta \epsilon}|_{\epsilon \approx 0} = \frac{\rho}{1-\alpha}$$

G.2 Constrained firms

Steady state growth. Constrained firms invest according to their maximum investment capacity which is capped by the net worth constraint.

$$\begin{aligned} k_{t+1,i} &= n_{t,i} + q_t b_{t+1,i} \\ &= n_{t,i} + q_t \xi n_{t,i} \\ &= (1 + q_t \xi)(z_t \theta_i k^\alpha_{t,i} + (1-\delta) k_{t,i} - b_{t,i}) \end{aligned}$$

Hence,

$$\begin{aligned} g_{cons} &= (1 + q_t \xi)(z_t \theta_i k^{\alpha-1}_{t,1} + (1-\delta) - b_{t,i}/k_{t,i}) \\ &= (1 + q_t \xi)(z_t \theta_i k^{\alpha-1}_{t,1} + (1-\delta) - \frac{\xi}{1 + q_{t-1}\xi}) \end{aligned}$$

Due to decreasing returns to scale, the growth rate is affected by the size of the firm, with larger firms growing slower

$$\frac{\Delta g_{cons}}{\Delta k_{t,i}} = (1+q_t\xi)(\alpha-1)z_t\theta_i k_{t,1}^{\alpha-2} < 0$$

For firms of the same size, those with a higher permanent productivity component grow quicker

$$\frac{\Delta g_{cons}}{\Delta \theta_i} = (1+q_t\xi)z_t k_{t,1}^{\alpha-1} > 0$$

Cyclicality Now consider the same setup as for unconstrained firms; at time $t=-1$, $z_t = z$. At time $t=0$, firms learn the future path of z_t, for $t \geq 0$ will be

$$z_t = z\exp(\rho^t \epsilon)$$

The growth rate on impact then becomes

$$g_{cons} = (1+q_t\xi)(z\exp(\rho^0\epsilon)\theta_i k_{t,1}^{\alpha-1} + (1-\delta) - \frac{\xi}{1+q_{t-1}\xi})$$

So, the elasticity of capital with respect to the shock ϵ is decreasing on capital and increasing on the productivity of the firm

$$\frac{\Delta g_{cons}}{\Delta \epsilon}|_{\epsilon\approx 0} = (1+q_t\xi)(z\theta_i k_{t,1}^{\alpha-1}) = \frac{(1+q_t\xi)}{\alpha} mpk_i$$

With the derivative of the elasticity with respect to the size and productivity of the firm being negative and positive respectively

$$\frac{\Delta^2 g_{cons}}{\Delta\epsilon\Delta\theta_i}|_{\epsilon\approx 0} = (1+q_t\xi)(zk_{t,1}^{\alpha-1}) > 0$$

$$\frac{\Delta^2 g_{cons}}{\Delta\epsilon\Delta k_{t,1}}|_{\epsilon\approx 0} = (\alpha-1)(1+q_t\xi)(z\theta_i k_{t,1}^{\alpha-2}) < 0$$

When is the elasticity of constrained larger than unconstrained?

$$\frac{\Delta g_{cons}}{\Delta \epsilon}|_{\epsilon\approx 0} > \frac{\Delta g_{uncons}}{\Delta \epsilon}|_{\epsilon\approx 0}$$

This happens when the marginal product of capital of constrained firms is above a given threshold

$$mpk > \rho \frac{\alpha}{1-\alpha} \frac{1}{1+q_t \xi}$$

So, two factors will determine which elasticity is larger: (i) the marginal product of capital of constrained firms, which depends on the distribution in terms of both size and productivity. The smaller and the more productive constrained firms are, the higher their elasticity; (ii) the persistence of the aggregate shock. As ρ approaches zero, unconstrained firms will not react to the shock, while the elasticity of constrained firms on impact does not depend on the persistence of the shock.