Online Appendix

Lumpy to Slumpy:
How Financing Costs drive Slow Recoveries

A Empirical analysis

A.1 Data construction

We build our sample from annual Compustat fundamentals for U.S. publicly listed firms between 1974 and 2024. Following the literature, we exclude financials (SIC 6000–6799), utilities (4900–4999), non-operating firms (9995), and conglomerates (9997), and restrict to firms incorporated in the United States. We further drop firm–years with non-positive assets, negative sales or capital expenditures, acquisitions 20% of lagged assets, leverage outside $[0, 10]$. To reduce the influence of outliers, we trim the top 0.1% of the investment rate distribution and winsorize other financial variables at the 0.5th and 99.5th percentiles.

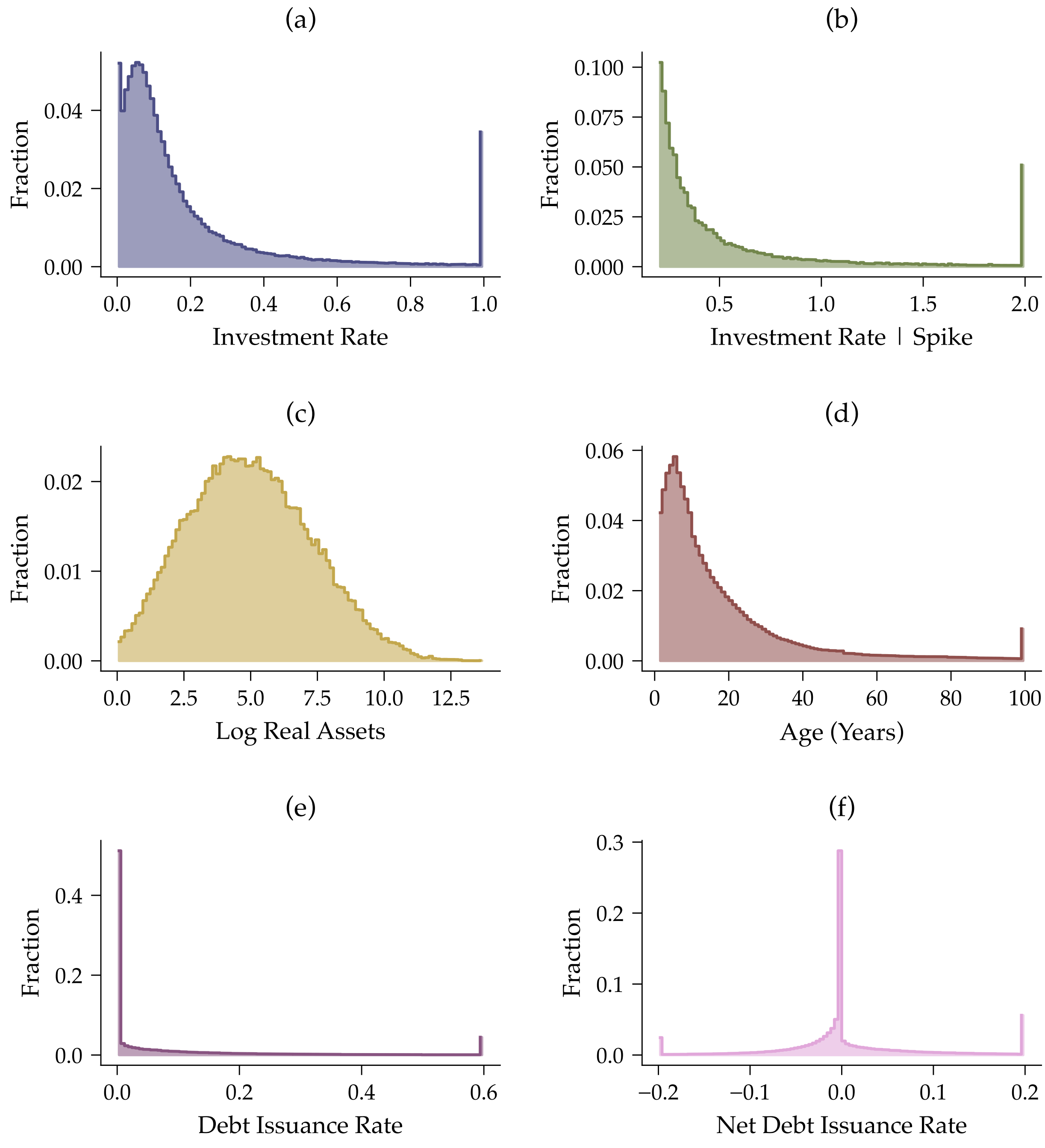
The main investment rate is defined as $\text{CAPX}/\text{PPEGT}_{t-1}$, with an investment spike whenever this rate exceeds 20%. We also define an inaction indicator for years with investment rates close to zero. Distance to default, our proxy for the external finance premium, is constructed from CRSP stock price data following Gilchrist & Zakrajšek (2012). We merge in FRED macro series and the Excess Bond Premium at the annual frequency, and construct debt issuance variables directly from Compustat flows.

Firm-level controls include leverage, liquidity, interest expenses, sales growth, size (log real assets), and age (from first appearance in Compustat and Datastream incorporation year). Industry dummies are based on major SIC groupings. For regressions, we use standardized versions of the cleaned and winsorized controls.

A.2 Summary statistics and distributions

Table A1 reports summary statistics for the final sample. Figure A1 shows the distributions of key observables, including the investment rate, spikes, age, size, and debt issuance measures. Extreme values are top- or bottom-coded at the thresholds reported in the caption.

Figure A1: Histograms for firm observables



Note: For readability, extreme values are top- or bottom-coded at the following thresholds: panel (a) at 1, panel (b) at 2, panel (d) at 100, panel (e) at 0.6, and panel (f) below 0.2 or above 0.4. Values beyond these limits are displayed as spikes at the respective thresholds.

Table A1: Summary Statistics

Variable	Mean	Median	P5	P95	Std. Dev.	N
Investment rate (%)	17.58	10.48	1.69	56.27	24.58	153465
Investment spike	0.24	0.00	0.00	1.00	0.43	153465
Net debt rate (%)	1.51	-0.02	-11.66	19.31	16.92	145673
Net debt spike	0.05	0.00	0.00	0.00	0.21	145673
Leverage (%)	28.39	24.08	0.00	73.21	28.05	153465
Distance to default	6.01	5.01	-0.20	15.59	5.83	109371
Firm age	19.85	12.00	2.00	68.00	21.35	153465
Firm size (log assets)	5.80	5.62	2.97	9.22	1.93	153453

Note: This table presents summary statistics for the main variables in the analysis. Investment rate is capital expenditures divided by lagged capital stock. Investment spike is an indicator for investment rate exceeding 20%. Net debt rate is the change in debt divided by lagged capital stock. Net debt spike indicates net debt issuance exceeding 5% of assets. Leverage is total debt divided by total assets. Distance to default is calculated following Merton (1974). Firm age is years since incorporation. Firm size is the natural logarithm of real total assets. The sample covers U.S. publicly traded firms from 1974-2024.

A.3 Further details on stylized fact 2: Financial position and investment concentration

To document our first stylized fact, we relate firms' average financial positions to measures of investment concentration. Specifically, for each firm i, we compute the time-series average of a given financial variable (e.g. liquidity, leverage, or distance to default) and correlate it with summary statistics of the firm's investment dynamics over time. This delivers a cross-sectional relationship between financial positions and investment lumpiness.

We now describe alternative measures for investment concentration at the firm level. Formally, let i_{jt} denote the investment rate of firm j in year t, observed over T_j years. We construct the following firm-level measures:

Gini coefficient. The Gini index measures inequality in the distribution of investment over time:

$$\text{Gini}_j = \frac{\sum_{t=1}^{T_j} \sum_{s=1}^{T_j} |i_{jt} - I_{js}|}{2T_j^2 \, \bar{i}_j},$$

where $\bar{i}_j$ is the mean of i_{jt} across t. A higher Gini coefficient means a lumpier investment profile.

Coefficient of Variation (CV). The coefficient of variation normalizes the standard deviation by the mean:

$$\mathrm{CV}_j = \frac{\sigma(i_{jt})}{\bar{i}_j}.$$

A higher CV coefficient proxies for a lumpier investment profile.

Herfindahl–Hirschman Index (HHI). As mentioned in the main text, the HHI captures the concentration of investment across active years:

$$\mathrm{HHI}_j = \sum_{t=1}^{T_j} \left(\frac{i_{jt}}{\sum_{s=1}^{T_j} i_{js}} \right)^2, \quad \text{where } i_{jt} > 0.$$

A higher HHI coefficient proxies for lumpier lumpy investment profile.

Kurtosis. Kurtosis measures the tail thickness of the investment distribution:

$$\mathrm{Kurtosis}_j = \frac{1}{T_j} \sum_{t=1}^{T_j} \left(\frac{i_{jt} - \bar{i}_j}{\sigma(i_{jt})} \right)^4.$$

Higher Kurtosis proxies for lumpier investment profiles.

Inaction share. We define inaction as periods with negligible investment:

$$\mathrm{Inaction}_j = \frac{1}{T_j} \sum_{t=1}^{T_j} \mathbf{1}\big(|i_{jt}| < 0.01\big).$$

Longer inaction means a lumpier investment profile.

Autocorrelation. We measure persistence in investment by the lag-1 autocorrelation:

$$\mathrm{Autocorr}_j = \frac{\sum_{t=2}^{T_j} (i_{jt} - \bar{i}_j)(i_{j,t-1} - \bar{i}_j)}{\sum_{t=1}^{T_j} (i_{jt} - \bar{i}_j)^2}.$$

Stronger autocorrelation means a lumpier investment profile.

These firm-level statistics summarize the temporal distribution of investment. Table A2 reports their correlations with average financial variables across firms, providing descriptive evidence on the link between financial frictions and investment lumpiness.

Table A2: Investment Concentration Measures and Financial Variables

Variable	Gini	CV	HHI	Kurtosis	Inaction	Autocorr
Liquidity	-0.061***	-0.128**	-0.052***	1.474***	-0.089***	-0.163***
	(0.015)	(0.053)	(0.011)	(0.446)	(0.015)	(0.031)
Distance to default	-0.007***	-0.015***	-0.003***	0.035**	-0.002	-0.002**
	(0.001)	(0.003)	(0.001)	(0.017)	(0.000)	(0.001)
Leverage	0.040***	0.086**	0.035***	-0.739**	0.031***	-0.023
	(0.011)	(0.034)	(0.006)	(0.307)	(0.009)	(0.023)
Interest coverage	0.125	0.322	0.200*	-3.931*	0.120	-0.233
	(0.115)	(0.313)	(0.116)	(2.142)	(0.081)	(0.159)

Note: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. The table shows the relationship between financial frictions and various measures of investment dispersion. Each column represents a different investment measure: Gini coefficient, coefficient of variation (CV), Herfindahl-Hirschman Index (HHI), kurtosis, inaction share, and autocorrelation.

A.4 Further details on stylized fact 3: Time-series of net debt spikes and investment spikes

In the main text we define an investment spike as an increase above 20% of lagged Property, Plant, and Equipment and net debt spike as an increase of above 20% of total lagged assets. Figure A2 replicates the same aggregate time series using alternative thresholds of 10% and 30%. The correlation between investment and net debt spikes remains strong in both cases, although it weakens somewhat at the 10% threshold, especially early in the sample when the investment series appears noisier.

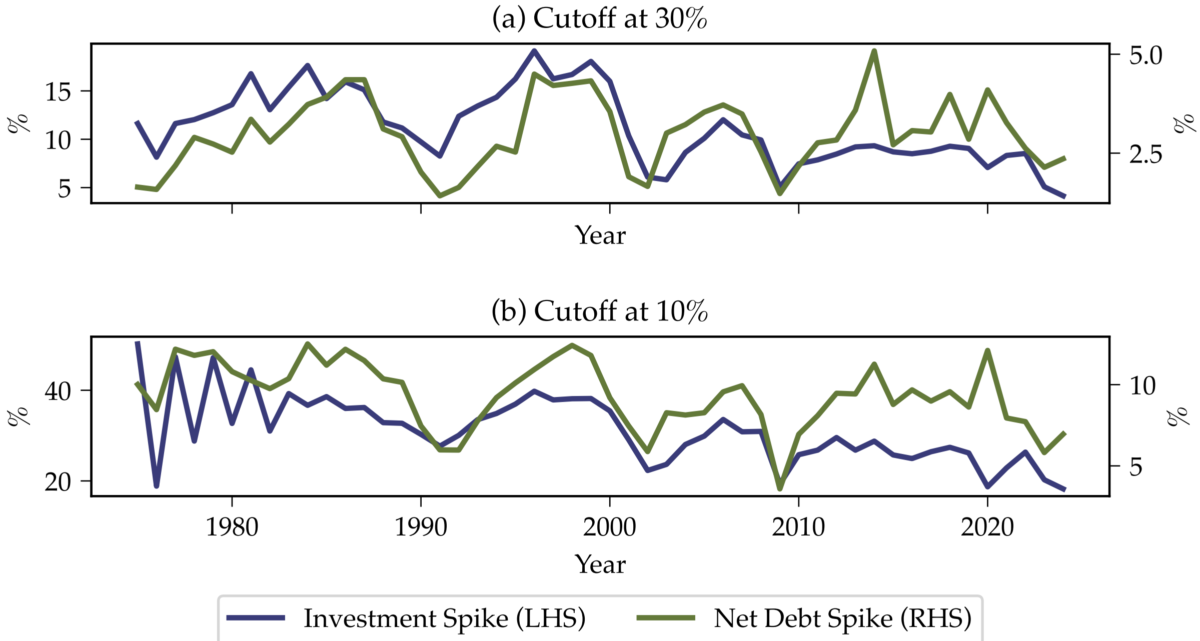
B Theoretical model

B.1 Bond Financing

Debt financing formulation This appendix shows how the reduced-form financing cost used in the main text can be written as an explicit debt-financing problem. Consider the version of the model with total financing needs

$$q(k', k) \equiv k' + \xi \, \mathbb{1}_{\{k' > (1-\delta+\nu)k\}}.$$

Figure A2: IRF of interest rate expenses to an investment spike.



Note: The right y-axis measures the percentage of firms with an investment spike in any given year whilst the left y-axis measures the percentage of firms with an net debt issuance spike in any given year.

Internal resources are $\psi(k,z) = \pi(k,z) + (1-\delta)k$. In the main formulation, the firm pays the average financing cost $\mathcal{R}(q(k',k),k,z)$ on total financing needs $q(k',k)$.

An equivalent debt-financing formulation makes the external claim explicit. The firm chooses capital and one-period debt b'_B according to

$$V^B(k,z) = \max_{k' \geq (1-\delta)k,\, b'_B \geq 0} \quad \psi(k,z) - q(k',k) + b'_B + \beta\left[\psi(k',z') - \mathcal{R}_B^{ext}(b'_B)b'_B\right]$$
$$\text{s.t.} \quad b'_B = \left[q(k',k) - \psi(k,z)\right]_+.$$

Thus the firm borrows only when total financing needs exceed internal resources.

Proposition A.1 (Debt-financing equivalence)**.**
Fix a state (k,z) and a feasible choice k'. Let

$$q \equiv q(k',k), \qquad b'_B \equiv q - \psi(k,z) > 0.$$

Conditional on external finance, $q > \psi(k,z)$, the main formulation and the debt-financing formulation deliver the same payoff at this k' if and only if

$$\beta \mathcal{R}_B^{ext}(b'_B; k,z)b'_B = \mathcal{R}(q,k,z)q - \psi(k,z).$$

Equivalently,

$$\mathcal{R}_B^{ext}(b'_B;k,z) = \frac{\mathcal{R}^{ext}(q,k,z)}{\beta}.$$

Proof.
Fix (k,z) and a feasible k' such that $q \equiv q(k',k) > \psi(k,z)$. Then external borrowing is

$$b'_B = q - \psi(k,z) > 0.$$

The main-formulation payoff from this same k' is

$$U^M(k') = \psi(k,z) - \mathcal{R}(q,k,z)q + \beta\psi(k',z').$$

This can be rewritten as

$$U^M(k') = \beta\psi(k',z') - \left[\mathcal{R}(q,k,z)q - \psi(k,z)\right].$$

The debt-financing payoff from the same k' and borrowing b'_B is

$$U^B(k') = \psi(k,z) - q + b'_B + \beta\left[\psi(k',z') - \mathcal{R}_B^{ext}(b'_B;k,z)b'_B\right].$$

Since $b'_B = q - \psi(k,z)$, the current-period term is zero:

$$\psi(k,z) - q + b'_B = 0.$$

Hence

$$U^B(k') = \beta\psi(k',z') - \beta\mathcal{R}_B^{ext}(b'_B;k,z)b'_B.$$

Equating $U^M(k') = U^B(k')$ gives

$$\beta\mathcal{R}_B^{ext}(b'_B;k,z)b'_B = \mathcal{R}(q,k,z)q - \psi(k,z).$$

This proves the first equivalence.

Finally, using the definition of the average financing cost in the external-finance region,

$$\mathcal{R}(q,k,z)q = \psi(k,z) + \mathcal{R}^{ext}(q,k,z)(q - \psi(k,z)),$$

so

$$\mathcal{R}(q,k,z)q - \psi(k,z) = \mathcal{R}^{ext}(q,k,z)b'_B.$$

Substituting into the equivalence condition and dividing by $b'_B > 0$ yields

$$\mathcal{R}_B^{ext}(b'_B; k, z) = \frac{\mathcal{R}^{ext}(q, k, z)}{\beta}.$$

If $q \leq \psi(k, z)$, then $b'_B = 0$ and both formulations reduce to the internal-finance case.

□

B.2 Micro-founded external finance cost

Micro-founded external finance cost This appendix relates the reduced-form financing schedule in the main text to a default-risk interpretation, as in Ottonello & Winberry (2020). Let $q(k', k)$ denote total financing needs. External financing needs are

$$b(q(k', k), k, z) \equiv \left[q(k', k) - \psi(k, z)\right]_+ .$$

The average unit cost of financing is

$$\mathcal{R}(q(k', k), k, z) = \begin{cases} 1 & \text{if } q(k', k) \leq \psi(k, z), \\ \dfrac{\psi(k, z)}{q(k', k)} + \mathcal{R}^{ext}(q(k', k), k, z)\left(1 - \dfrac{\psi(k, z)}{q(k', k)}\right) & \text{if } q(k', k) > \psi(k, z). \end{cases} \tag{A.1}$$

Suppose the external financing cost is generated by default risk:

$$\mathcal{R}^{ext}(q(k', k), k, z) = \frac{1}{1 - \mathbb{E}(\chi_{t+1} LGD_{t+1} \mid q(k', k), k, z)}, \tag{A.2}$$

where χ_{t+1} is a default indicator and LGD_{t+1} is loss given default.

Proposition A.2 (Mapping e_0)**.**
Let

$$L_0 \equiv \mathbb{E}\left(\chi_{t+1} LGD_{t+1} \mid q(k', k) = \psi(k, z), k, z\right).$$

The reduced-form e_0 is the external-finance unit cost at the internal-finance boundary, so that $e_0 - 1$ is the corresponding wedge:

$$e_0 = \frac{1}{1 - L_0}, \qquad e_0 - 1 = \frac{L_0}{1 - L_0}.$$

Proof.

Let

$$L_0 \equiv \mathbb{E}\big(\chi_{t+1} LGD_{t+1} \mid q(k',k) = \psi(k,z), k, z\big)\,.$$

Below the internal-finance boundary, the marginal cost of funds is one. From the right, as external financing needs become infinitesimal, the marginal cost is

$$\lim_{q(k',k)\to\psi(k,z)^+} MC = \frac{1}{1-L_0}.$$

The reduced-form schedule in the main text has right limit e_0. Therefore,

$$e_0 = \lim_{q(k',k)\to\psi(k,z)^+} MC = \frac{1}{1-L_0}.$$

Moreover,

$$e_0 - 1 = \lim_{q(k',k)\to\psi(k,z)^+} MC - \lim_{q(k',k)\to\psi(k,z)^-} MC = \frac{L_0}{1-L_0}.$$

□

If a firm has positive default risk even when external financing needs are infinitesimal, the marginal cost curve has a discontinuity at the internal-finance boundary and $e_0 > 1$.

Remark 1 (Mapping e_1)**.**
Let

$$L(b,k,z) \equiv \mathbb{E}(\chi_{t+1} LGD_{t+1} \mid b, k, z)\,, \qquad b = q(k',k) - \psi(k,z) > 0.$$

Then the slope of the external-finance schedule with respect to external financing needs is

$$\frac{\partial \mathcal{R}^{ext}}{\partial b} = \frac{\partial L(b,k,z)/\partial b}{[1 - L(b,k,z)]^2}.$$

Thus the reduced-form parameter e_1 captures the local sensitivity of default risk or loss given default to external financing needs.

In the logic of Ottonello & Winberry (2020), higher external financing raises default risk or loss given default. Larger external financing needs, $q(k',k) - \psi(k,z)$, therefore map into higher external financing costs, which is the force summarized by e_1 in the reduced-form schedule.

B.3 Productivity-threshold characterization

The main text characterizes the extensive-margin adjustment decision in terms of the largest fixed adjustment cost the firm is willing to pay. The same logic can be stated in terms of a productivity threshold. Fix (k, z, ξ) and let

$$D(z'; e_0, e_1) \equiv V^A(k, z, z', \xi; e_0, e_1) - V^{NA}(k, z, z'; e_0, e_1)$$

denote the payoff gain from adjusting.

Proposition A.3 (External finance costs raise the productivity threshold)**.**
Let z^* denote the indifference productivity threshold, implicitly defined by

$$D(z^*; e_0, e_1) = 0.$$

Then the firm adjusts iff $z' \geq z^*$. Higher external-finance costs raise the productivity threshold:

$$\frac{\partial z^*}{\partial e_i} \geq 0, \qquad i \in \{0, 1\}.$$

The inequality is strict whenever the marginal adjusting firm uses external finance.

Proof.
First, D is strictly increasing in z'. The only direct effect of z' is through future net worth $\beta\psi(k', z')$. By the envelope theorem,

$$\frac{\partial V^A}{\partial z'} = \beta(k'_A)^\alpha, \qquad \frac{\partial V^{NA}}{\partial z'} = \beta(k'_{NA})^\alpha,$$

where k'_A and k'_{NA} denote the optimal capital choices under adjustment and non-adjustment, respectively. Adjustment requires

$$k'_A > (1 - \delta + \nu)k,$$

whereas non-adjustment requires

$$k'_{NA} \leq (1 - \delta + \nu)k.$$

Therefore, $k'_A > k'_{NA}$ and, since $\alpha > 0$,

$$\frac{\partial D}{\partial z'} = \beta\left[(k'_A)^\alpha - (k'_{NA})^\alpha\right] > 0.$$

Thus D crosses zero at most once, so the adjustment decision is characterized by the productivity cutoff z^*. Let

$$b_A \equiv \left[q(k'_A, k) - \psi(k,z)\right]_+, \qquad b_{NA} \equiv \left[k'_{NA} - \psi(k,z)\right]_+$$

denote external financing needs under adjustment and non-adjustment, evaluated at their respective optimal choices. Since $q(k'_A, k) = k'_A + \xi$ and $\xi \geq 0$, while $k'_A > k'_{NA}$, we have

$$q(k'_A, k) > k'_{NA}.$$

Hence

$$b_A \geq b_{NA},$$

with strict inequality whenever the adjustment option uses external finance. By the envelope theorem,

$$\frac{\partial V^j}{\partial e_0} = -b_j, \qquad \frac{\partial V^j}{\partial e_1} = -b_j^2, \qquad j \in \{A, NA\}.$$

Therefore,

$$\frac{\partial D}{\partial e_i} = \frac{\partial V^A}{\partial e_i} - \frac{\partial V^{NA}}{\partial e_i} \leq 0, \qquad i \in \{0, 1\},$$

with strict inequality whenever the marginal adjusting firm uses external finance. At the productivity threshold,

$$D(z^*; e_0, e_1) = 0.$$

By the implicit function theorem,

$$\frac{\partial z^*}{\partial e_i} = -\frac{\partial D/\partial e_i}{\partial D/\partial z'}.$$

Since $\partial D/\partial e_i \leq 0$ and $\partial D/\partial z' > 0$, it follows that

$$\frac{\partial z^*}{\partial e_i} \geq 0.$$

The inequality is strict whenever the marginal adjusting firm uses external finance. □

C Model appendix

C.1 Numerical Solution Algorithm

This section explains the implementation of the global sequence-space solution method, the repeated transition method (RTM) developed in Lee (2026), which we use to compute the recursive competitive equilibrium of Section 4.

The model generates potentially highly nonlinear aggregate dynamics: the extensive margin of lumpy capital adjustment interacts with the endogenous external-finance wedge, and both margins depend on the cross-sectional distribution of firms over capital and productivity. One of the most challenging steps in obtaining a global solution for this class of models lies in specifying the correct law of motion for the endogenous aggregate state: the law of motion can be correctly specified only when the solution is known, while the solution can be obtained only after the law of motion is specified. Approximating the law of motion with a parametric forecasting rule presumes a particular (typically log-linear) shape of the aggregate dynamics, which is restrictive in our context, as the state dependence of recovery dynamics is the object of interest rather than a property to be imposed a priori.

The repeated transition method overcomes this hurdle by exploiting the ergodicity of the recursive competitive equilibrium. If the simulated path of the economy is long enough, the equilibrium allocations relevant for the ergodic dynamics are realized on the path. The rationally expected state-contingent future outcomes can then be constructed by locating, for each possible future exogenous state, periods on the simulated path whose endogenous aggregate state brackets the conjectured future endogenous state—without specifying a parametric law of motion. The method iterates directly on the sequences of equilibrium prices and aggregate allocations along the simulated path.

Algorithm. Let $X \equiv (A, \bar{\xi}, e_0)$ denote the exogenous aggregate state. The three shock processes are independent AR(1) processes discretized by the Tauchen method, with three grid points for A and two grid points each for $\bar{\xi}$ and e_0, so that X lives on a joint grid of $3 \times 2 \times 2 = 12$ states whose transition matrix $\mathcal{P}_{X,X'}$ is the Kronecker product of the marginal transition matrices. Since the firm distribution Φ evolves deterministically given S, the matrix $\mathcal{P}_{X,X'}$ is the stochastic kernel of the aggregate transition measure $\mathcal{P}(S'|S)$ in the household problem.

1. Simulate a T-period path of the exogenous aggregate state $\{X_t\}_{t=1}^{T}$ with $T = 1{,}601$; the first and last $B = 100$ periods of the path serve as buffers that are excluded from

the state-matching step below.

2. For the ith iteration, guess the time series of the marginal utility of consumption and aggregate capital,

$$\{p_t^{(i)}\}_{t=1}^{T}, \quad \{K_t^{(i)}\}_{t=1}^{T+1}, \qquad p_t \equiv 1/C_t,$$

   where the capital path includes the period-$(T+1)$ terminal element. We use the pre-computed stationary-equilibrium levels as the initial guess for all periods, with an infinitesimal perturbation of the capital path that breaks exact symmetry across periods in the first iteration. Given $p_t^{(i)}$, the wage follows from the household intratemporal condition, $w_t^{(i)} = \eta / p_t^{(i)}$, and the stochastic discount factor between period t and a successor exogenous state X' is $\beta\, p^{(rv)}(X')/p_t^{(i)}$ by Equation (40), where the superscript (rv) indicates that the allocation is a random variable over next-period exogenous states.

3. *(Backward)* Solve the firm problem backward from the terminal period. For each t, the exogenous state X_t sets the TFP level A_t, the fixed-cost bound $\bar{\xi}_t$, and the financing-cost intercept $e_{0,t}$. Given $(w_t^{(i)}, p_t^{(i)})$ and the state-contingent continuation value constructed as described below, we compute on the (k, z) grid: the static labor demand and operating profit; internal funds $\psi_t(k, z)$; the financing-cost fixed point $R_t^*(k, z)$ of Equation (35), which is solved exactly on the grid of candidate capital choices; the adjustment target $k_t^*(k, z)$; the constrained choice $k_t^c(k, z) \in \Omega(k)$; and the threshold draw $\xi_t^*(k, z)$ at which the firm is indifferent between paying the fixed cost and remaining within the constrained adjustment set. Since $G(\cdot; \bar{\xi}_t)$ is uniform on $[0, \bar{\xi}_t]$ in our calibration, the adjustment probability is $h_t(k, z) = G\big(\xi_t^*(k, z); \bar{\xi}_t\big) = \xi_t^*(k, z)/\bar{\xi}_t$. This step delivers the conditionally optimal value functions and policies $\{J_t^*, k_t^*, k_t^c, \xi_t^*\}_{t=1}^{T}$.

4. *(Forward)* Starting from the stationary distribution, simulate the cross-sectional distribution Φ_t forward using the period-t policies and the nonstochastic histogram method of Young (2010): at each (k, z), the firm's mass is split between the adjustment branch (weight h_t) and the constrained branch (weight $1 - h_t$), and each branch's next-period capital is assigned to the two neighboring grid points with lottery weights. Aggregation delivers the conditionally optimal sequences of capital $K_{t+1}^* = \int k' \, d\Phi_t$, output, investment, and labor; goods-market clearing then implies consumption C_t^* and the implied marginal utility $p_t^* = 1/C_t^*$.

5. Check convergence: the mean of the squared deviations pooled across the price and

capital paths must fall below the tolerance,

$$\text{tol} \; > \; \frac{1}{2T+1}\left[\sum_{t=1}^{T}\left(p_t^* - p_t^{(i)}\right)^2 + \sum_{t=1}^{T+1}\left(K_t^* - K_t^{(i)}\right)^2\right],$$

with $\text{tol} = 10^{-6}$; the capital path includes the terminal element.

6. If the inequality is satisfied, the ith guess is the solution. Otherwise, update the guess by the convex combinations

$$K_t^{(i+1)} = \omega_K K_t^{(i)} + (1-\omega_K)K_t^*, \qquad p_t^{(i+1)} = \omega_p\, p_t^{(i)} + (1-\omega_p)p_t^*,$$

with weights on the previous iteration $\omega_K = 0.99$ and $\omega_p = 0.997$, and return to step 3.

State-contingent expectations. In step 3, the expectation in the firm's continuation value requires the value function and prices at every possible successor exogenous state X', whereas the simulated path realizes only one successor at $t+1$. For the realized successor X_{t+1}, the period-$(t+1)$ objects on the path are used directly. For each counterfactual $X' \neq X_{t+1}$, we exploit the ergodicity of the simulated path:

3-1. Form a partition of the path based on the exogenous-state realizations,

$$\mathcal{T}^{X'} = \{\tau \,:\, X_\tau = X'\},$$

excluding the buffer periods at both ends of the path.

3-2. Within $\mathcal{T}^{X'}$, find the two periods τ_l and τ_h whose aggregate capital stocks are closest to the conjectured $K_{t+1}^{(i)}$ from below and from above, respectively. Aggregate capital is used as the matching statistic for the endogenous aggregate state: it is the first moment of Φ and the payoff-relevant endogenous aggregate allocation that evolves dynamically.

3-3. Construct the state-contingent objects by linear interpolation. In the implementation, the value function carried on the path is measured in marginal-utility units, $\tilde{J}_\tau \equiv p_\tau J_\tau$, so that the stochastic discount factor of Equation (40) is embedded in the denomination of the value function rather than applied as a separate factor. With weight

$$\omega_l = \frac{K_{\tau_h} - K_{t+1}^{(i)}}{K_{\tau_h} - K_{\tau_l}} \in [0,1],$$

the contingent value function and marginal utility are

$$\tilde{J}^{(rv)}(\cdot,\cdot;X') = \omega_l\,\tilde{J}_{\tau_l}(\cdot,\cdot) + (1-\omega_l)\,\tilde{J}_{\tau_h}(\cdot,\cdot), \qquad p^{(rv)}(X') = \omega_l\,p_{\tau_l} + (1-\omega_l)\,p_{\tau_h},$$

and the continuation value of a firm choosing capital k' with current productivity z is computed as

$$\mathbb{E}\left[m\,J(k',z',S')\,\middle|\,z,X_t\right] = \frac{\beta}{p_t^{(i)}}\sum_{X'}\mathcal{P}_{X_t,X'}\,\mathbb{E}_{z'|z}\left[\tilde{J}^{(rv)}(k',z';X')\right],$$

which implements $\mathbb{E}[m\,J']$ with $m = \beta\,p'/p$ exactly. The risk-free rate on the path is computed analogously from the expected marginal utility, $1+r_t = (1/\beta)\,p_t^{(i)}/\mathbb{E}_t[p^{(rv)}]$.

Convergence and accuracy. The fixed point is reached after approximately 500 damped outer-loop iterations at the baseline calibration. The convergence of the method hinges on the stability and uniqueness of the equilibrium. Beyond the fixed-point tolerance, we verify the dynamic consistency of the converged solution: at every period of the path, the next-period aggregate capital implied by aggregating the period-t policies over Φ_t must coincide with the realized capital on the path. At convergence, the maximum absolute deviation is 0.18% of steady-state capital, the root-mean-squared deviation is 0.018%, and the mean signed deviation is below 0.001% in absolute value, indicating no systematic bias in the expectations implied by the state-matching step.

C.2 Numerical Calibration

This appendix complements the calibration discussion in Section 4. It makes precise the construction of three model-side moments in Table 3: the cross-sectional standard deviation of investment rates, the implied stock leverage ratio, and the external-finance spread. These moments require resolving the mixture between adjusting and non-adjusting firms, or mapping an intra-period financing margin into an empirical stock object. Table C1 presents the parameters fixed outside the model.

Throughout, let $\Phi(k,z)$ denote the stationary distribution over firm states, $h(k,z)$ the equilibrium probability that a firm adjusts at state (k,z), and $k^A(k,z)$ and $k^N(k,z)$ the capital choices of adjusting and non-adjusting firms. Let $b^A(k,z)$ denote the external-finance choice of an adjusting firm. The corresponding investment rates are

$$\iota^A(k,z) = \frac{k^A(k,z)-(1-\delta)k}{k}, \qquad \iota^N(k,z) = \frac{k^N(k,z)-(1-\delta)k}{k}.$$

Table C1: Fixed parameters

Parameter	Symbol	Value
Capital share	α	0.256
Labor share	γ	0.640
Depreciation	δ	0.090
Discount factor	β	0.977
Labor disutility	η	2.400
Internal-resource share of undepreciated capital	θ	1.000
Idiosyncratic persistence	ρ_z	0.859
Idiosyncratic volatility	σ_z	0.022

The model moments are computed from the stationary distribution and policy functions. As a check, the same moments are also computed from a simulated firm panel, and the two computations agree.

Cross-sectional standard deviation of the investment rate. The data target is the cross-sectional standard deviation of i/k for U.S. publicly listed firms in Ottonello & Winberry (2020). The model counterpart is the standard deviation under the two-branch mixture induced by the adjustment decision. The unconditional mean investment rate is

$$\bar{\iota} = \sum_{k,z} \Phi(k,z) \left[h(k,z)\iota^A(k,z) + (1 - h(k,z))\, \iota^N(k,z) \right].$$

The standard deviation is then

$$\mathrm{sd}(i/k) = \left(\sum_{k,z} \Phi(k,z) \left[h(k,z) \left(\iota^A(k,z) - \bar{\iota} \right)^2 + (1 - h(k,z)) \left(\iota^N(k,z) - \bar{\iota} \right)^2 \right] \right)^{1/2}.$$

The statistic is computed on the support of Φ, with no trimming.

Implied stock leverage. The model does not carry debt as a state variable; external finance is intra-period. Therefore, no model object directly corresponds to empirical stock leverage. Several flow-based quantities are well-defined, including the conditional borrowing ratio of adjusting borrowers, the aggregate per-period external-finance flow per unit of capital, and the external-finance share of investment. To map the model into an empirical stock leverage ratio, we combine the aggregate external-finance flow with an empirical debt amortization rate.

Let

$$\frac{B}{K} = \frac{\sum_{k,z} \Phi(k,z)h(k,z)b^A(k,z)}{\sum_{k,z} \Phi(k,z)k}$$

denote aggregate per-period external finance per unit of installed capital. We then construct

$$\frac{D}{K} = \frac{B/K}{\delta_D}.$$

In the calibration, $B/K = 0.047$. We set $\delta_D = 1/7 \simeq 0.143$, corresponding to a seven-year average debt maturity, which is the median bond maturity at issuance reported by Choi, Hackbarth, and Zechner (2018) for U.S. firms in the Capital IQ sample from 2002 to 2012. This gives

$$D/K = \frac{0.047}{0.143} = 0.329,$$

against the empirical target of 0.340.

External-finance spread. The external-finance spread is the borrower-weighted average external-finance wedge in the stationary equilibrium. For an adjusting firm that borrows $b^A(k,z) > 0$, the marginal external-finance cost is

$$R^{ext}(k,z) = e_0 + e_1 b^A(k,z).$$

Let

$$\omega(k,z) = \Phi(k,z)h(k,z)\mathbb{1}\{b^A(k,z) > 0\}$$

be the stationary mass of borrowing firms at state (k,z). The model spread is

$$\text{spread}(\%) = 100 \times \frac{\sum_{k,z} \left(R^{ext}(k,z) - 1\right) \omega(k,z)}{\sum_{k,z} \omega(k,z)}.$$

The data target is the firm-level external-finance spread in Ferreira et al. (2024), equal to 2.04%. The calibrated model delivers 2.10%.

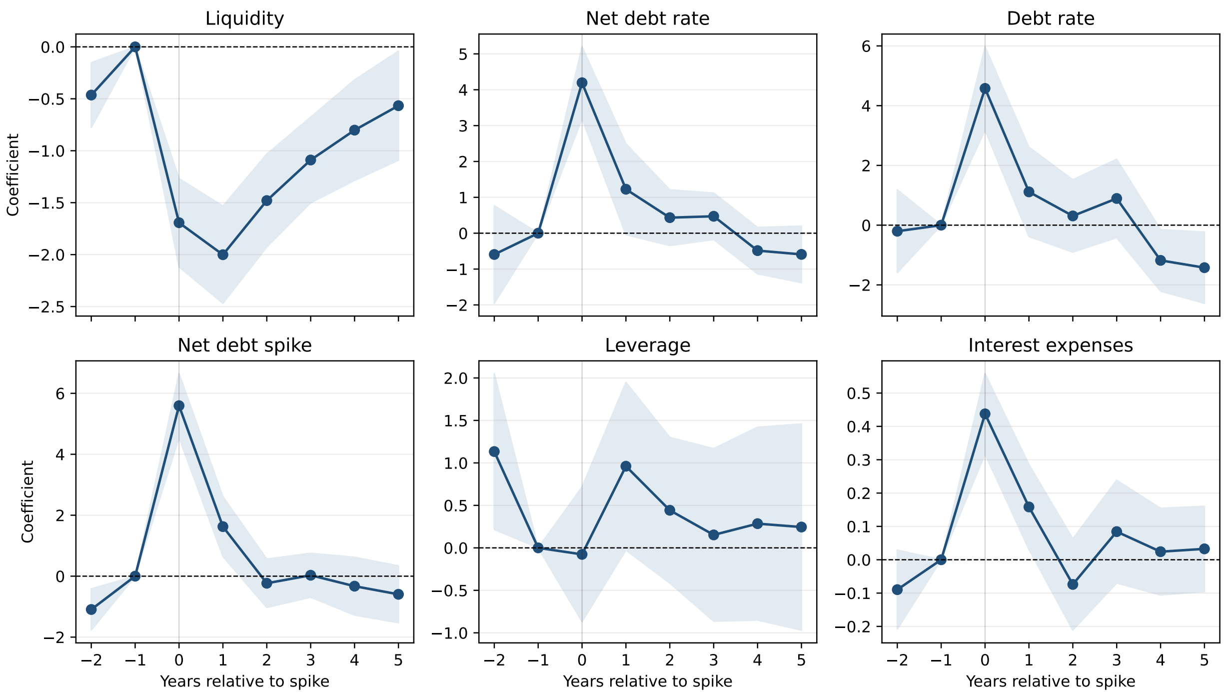
D Mechanism validation robustness

D.1 Robustness for investment spikes and financial conditions

This subsection reports additional results for Section 6.1. It first extends the local-projection exercise to financial outcomes beyond distance to default. It then reports the duration-based bonus-depreciation exercises that support the first stage of the tax design.

The local projections in Figure D1 use the same specification as Equation (42). The figure shows that isolated investment spikes are associated with a decline in liquidity and contemporaneous increases in debt issuance, the probability of a debt issuance spike and interest expenses. Together with the main distance-to-default response, these auxiliary outcomes point to a broader deterioration in firms' financial positions after large investment episodes.

Figure D1: Additional financial outcomes after isolated investment spikes

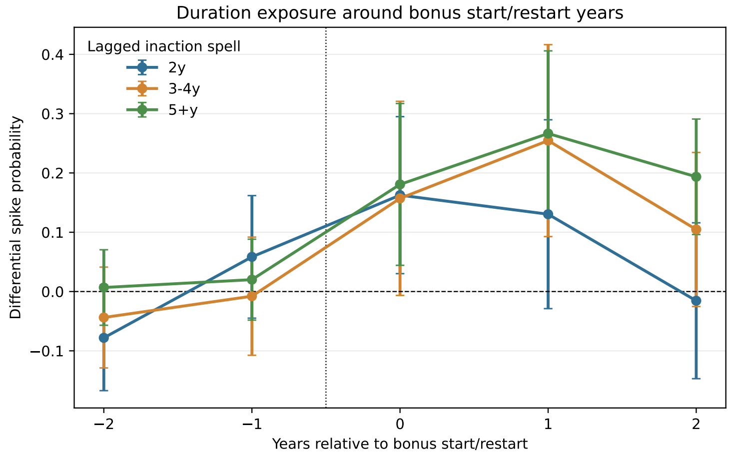


Notes: The figure reports local-projection coefficients from Equation (42) for additional financial outcomes. Shaded areas are 95 percent confidence intervals.

Figure D2 estimates an event-study specification around the 2001 introduction of bonus depreciation and its 2008 reintroduction. The exposure variable is lagged inaction length. This measure starts counting only after a previously observed spike: the first observed spike is not used to measure the length of an inaction spell because the previous spike may have occurred before the firm enters Compustat, so the exact number of periods since the last spike is unobserved. For firms in five-or-more-year inaction spells, the coefficients are close to zero in the two pre-periods, then rise sharply in the start year and remain positive in the following two years. Bonus depreciation therefore triggers spikes primarily among firms with accumulated adjustment pressure.

Figure D3 reports a related IV robustness exercise based directly on lagged inaction duration. Let $\text{Start}_{t,t+1}$ equal one in bonus-depreciation start/restart years and the fol-

Figure D2: Event study around bonus-depreciation start/restart years



Notes: The figure plots event-time coefficients for the interaction between lagged inaction-duration bins and indicators for years around the 2001 and 2008 bonus-depreciation start/restart episodes. The dependent variable is the investment-spike dummy, estimated conditional on beginning the year in an inaction spell. The omitted duration bin is one year since the last spike. Vertical bars are 95 percent confidence intervals.

lowing year. The excluded instrument is

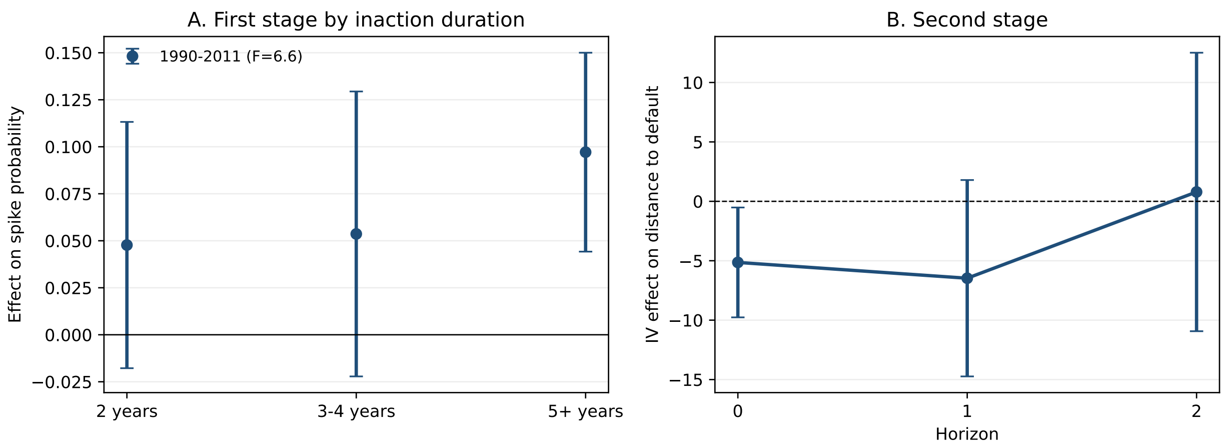
$$Z_{it}^{D,b} = \mathbf{1}\{\text{Duration bin}_{i,t-1} = b\} \times \text{Start}_{t,t+1}, \qquad b \in \{2,\ 3\text{–}4,\ 5+\}.$$

The omitted duration bin is one year since the last spike. This is not the baseline capital-intensity instrument in the main text, but it is a useful model-based robustness check because firms further into an inaction spell should be closer to an adjustment trigger. The first stage results are stronger for firms in longer inaction spells. The impact of an increased spike probability induced by the policy on distance to default is −5.14 and statistically significant at the five percent level. The one-year coefficient is also negative but less precise, and the two-year coefficient is close to zero.

D.2 Robustness for financial constraints and spike probabilities

This subsection reports robustness results for Section 6.2. The figures repeat the linear probability model in Equation (44) using alternative measures of financial constraints. In each panel, the plotted estimates are the coefficients θ_{db} on the interactions between lagged financial-constraint deciles and years since the last investment spike. The horizontal axis ranks firms by lagged financial constraints, with decile 10 denoting the most constrained firms. The vertical bars are 95 percent confidence intervals.

Figure D3: Inaction-duration robustness: first and second stages



Notes: Panel A reports first-stage coefficients from regressions of the investment-spike dummy on interactions between lagged inaction-duration bins and a bonus-depreciation start/restart-window dummy. Panel B reports IV local-projection coefficients for distance to default, where the investment-spike dummy is instrumented with the duration-bin interactions shown in Panel A. The sample is restricted to firm-years that begin in an inaction spell. Vertical bars are 95 percent confidence intervals.

Figure D4 shows that the negative association between financial weakness and investment spikes is not unique to distance to default. Across inaction-duration bins, the estimated coefficients generally decline as firms become less liquid, more levered, or have lower cash flow. Thus, using alternative measures of balance-sheet strength delivers the same qualitative pattern: firms are less likely to undertake investment spikes when their financial positions are weaker.

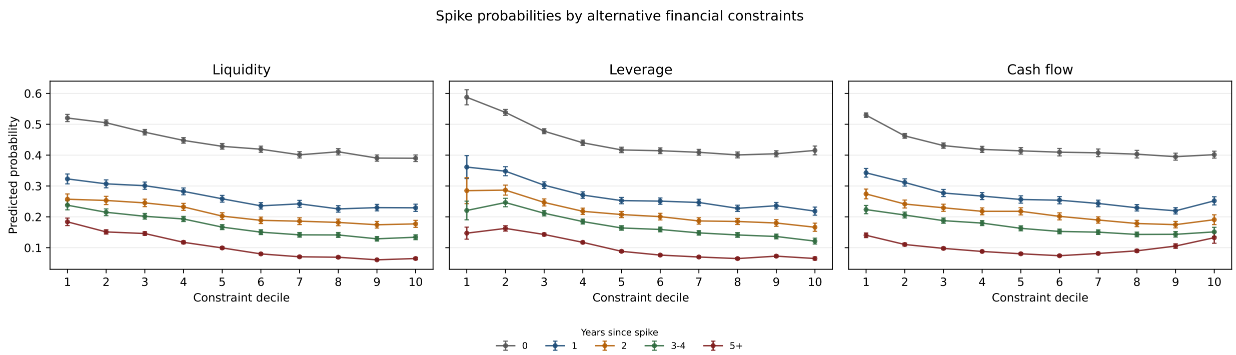
D.3 Robustness for pre-recession firm distributions

The main text measures the dispersion of non-adjustment age using investment rates below 3.78 percent to define non-adjustment years. Table D1 repeats this moment using the stricter 1 percent threshold. The variance of non-adjustment age remains higher before weak recoveries than before historical non-weak recoveries.

References

Ferreira, T. R. T., Ostry, D. A., & Rogers, J. (2024). *Firm financial conditions and the transmission of monetary policy*. Bank of England working papers 1093, Bank of England.

Figure D4: Investment-spike coefficients by alternative financial constraints



Notes: The figure reports estimates of the coefficients θ_{db} from the linear probability model in Equation (44), using lagged liquidity, leverage, and cash-flow deciles as alternative financial-constraint measures. Deciles are oriented so that higher values indicate tighter financial conditions: lower liquidity, higher leverage, and lower cash flow. The colored lines correspond to years since the last investment spike. Vertical bars report 95 percent confidence intervals.

Table D1: Pre-recession dispersion in non-adjustment age: 1 percent threshold

Recovery group	Var(a)
Non-weak recoveries	0.02
Weak recoveries	0.05

Notes: The table averages recession-level moments separately for weak and historical non-weak recoveries. Weak recoveries are 1990–91, 2001, and 2007–09. Historical non-weak recoveries are 1969–70, 1973–75, and 1980–82. The 2020 pandemic recession is classified as a non-weak recovery but is excluded. Let $i_{it} \equiv \text{CAPX}_{it}/\text{PPEGT}_{i,t-1}$. For this robustness exercise, the adjustment dummy is $\text{Adj}^1_{it} \equiv \mathbf{1}\{i_{it} \geq 0.01\}$, and a is the number of years since the last observed year with $\text{Adj}^1_{it} = 1$.

Gilchrist, S. & Zakrajšek, E. (2012). Credit spreads and business cycle fluctuations. *American Economic Review*, 102(4), 1692–1720.

Lee, H. (2026). A Global Dynamic Nonlinear Solution Framework and the Repeated Transition Method. *Working paper*.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 29(2), 449–470.

Ottonello, P. & Winberry, T. (2020). Financial heterogeneity and the investment channel of monetary policy. *Econometrica*, 88(6), 2473–2502.

Young, E. R. (2010). Solving the incomplete markets model with aggregate uncertainty using the krusell–smith algorithm and non-stochastic simulations. *Journal of Economic Dynamics and Control*, 34(1), 36–41.